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A simplified analytical model for metal sandwich beam with soft core under impulsive loading over a central patch

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ABSTRACT

A simplified analytical model is proposed to predict the large deflection response of fully clamped metal sandwich beam with soft core subjected to impulsive loading over a central patch. Using the classical yield condition for sandwich cross-section, the dynamic response is uncoupled into two distinct phases: a bending-only phase and a plastic string phase. Analytical solutions for the metal sandwich beams under impulsive loading are then obtained. Comparisons of the analytical predictions with finite element results manifest that the simplified analytical model can accurately capture the dynamic response of sandwich structure with a soft core. Finally, the analytical formulae are employed for the minimum mass design of sandwich structures.

1. Introduction

Sandwich structure has been paid more attention due to its excellent property of withstanding the extreme blast loadings. However, the advantages of such a structural system depend on the innovative topological design and manufacture of cellular cores [1–4]. Then, various low density metal cores have been developed to meet the functional requirements in engineering applications, such as metal foams [2], hexagonal and square honeycombs [5,6], lattice materials of pyramidal and tetrahedral arrangements [4,7], egg-box [8], woven material [9] and functionally graded [10–12] and hierarchical cores [13,14].

Over the last decade, dynamic response of metal sandwich structures has been investigated extensively. Fleck and Deshpande [15] developed an analytical model to study the dynamic response of the fully clamped sandwich beam subjected to uniform blast loading. Subsequently, Qiu et al. [16] developed an analytical model for the dynamic response of fully clamped sandwich beam subjected to blast loading over a central patch. Using the inscribing and circumscribing squares of the classical yield criterion for sandwich structures, the so-called upper and lower ‘bounds’ were given for the dynamic response of the sandwich beams subjected to blast loading. The parallel investigations performed by Xue and Hutchinson [17] and Ebrahimi and Vaziri [18] showed that optimized sandwich structures outperform solid counterparts with the same mass under shock loading. Tilbrook et al. [19] proposed an analytical lumped mass model based on the relative time-scales of compression and the combination of plastic bending and longitudinal stretching, and conducted finite element analysis to classify the impulsive response of sandwich beam. Considering the effect of fluid–structure interaction, the dynamic response of metal sandwich structures was investigated also [20–24]. Employing a shock simulation technique involving high-speed impact of aluminum foam projectiles, the dynamic response of the fully clamped sandwich beam was experimentally investigated [25–27]. Recently, Qin and Wang [28] derived a new yield criterion for the metal sandwich structures including the effect of core strength. Using the new yield criterion, Qin and Wang [29] and Qin et al. [30] obtained analytical solutions for the impulsive response of fully clamped metal sandwich beams by using the membrane factor method [31,32], in which the interaction of bending and stretching is considered. Also, the new 'bounds' of the solutions were derived on the basis of the new yield criterion, which are tighter than those based on the classical yield criterion.

Using the framework for the fully clamped sandwich beams proposed by Fleck and Deshpande [15], Qiu et al. [33] developed an analytical model for dynamic response of fully clamped circular sandwich plates. Neglecting the effect of bending moment, Qin and Wang [34] obtained the membrane mode solutions for the impulsive response of fully clamped circular sandwich plates with metal foam core, which is simple and available for engineering

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application. Using the new yield criterion considering the effect of core strength [28], Zhu et al. [35] gave the lower and upper 'bounds' of the maximum central deflection and the response time for fully clamped square sandwich plates subjected to air blast loading. Using the classical yield criterion [36], Cui et al. [37] developed an analytical model for the dynamic response of fully clamped square lattice sandwich plates subjected to air blast loading. Liu et al. [38] developed an analytical mo0del for the elastic–plastic dynamic response of fully backed sandwich plates under localized impulsive loadings, in which the core is modeled as an elastic-perfectly plastic foundation. The parallel experimental investigations on the dynamic response of fully clamped sandwich plates with various cores subjected to air blast loading were performed also [39–42].

The objective of this work is to develop a simplified analytical model for the dynamic response of fully clamped metal sandwich beams with soft core subjected to a localized impulsive loading. This paper is organized as follows. Firstly, using the classical yield condition for the sandwich cross-section, a critical midspan deflection is obtained for decoupling this problem. Next, an analytical model is developed for the dynamic response of fully clamped metal sandwich beams under impulsive loading over a central patch. Finally, finite element (FE) analysis for the dynamic response of the sandwich beams is performed and compared with the theoretical results.

2. Problem formulation

Consider a fully clamped metal sandwich beam subjected to impulsive loading I per unit length over a central patch of length $2a$, as shown in Fig. 1. Length of the sandwich beam is $2L$, thickness of the identical face sheets is h and thickness of the foam core is c. It is assumed that the face sheets are made from rigid-perfectly plastic material with yield strength σ_f and density ρ_f, as shown in Fig. 2(a). The foam core is modeled as a rigid-perfectly plastic locking (r-p-p-l) material, as shown in Fig. 2(b). The parameters ρ_c, σ_n, σ_l and ε_D are denoted the density, normal compressive strength, longitudinal strength and densification strain of the foam core, respectively.

Fleck and Deshpande [15] decoupled the dynamic response of core compression and plastic bending and stretching. In the core compression stage, they assumed a one-dimensional slice through the thickness of the sandwich beam and neglected the reduction in momentum due to the impulse provided by the fully clamped supports. In what follows, these assumptions are adopted.

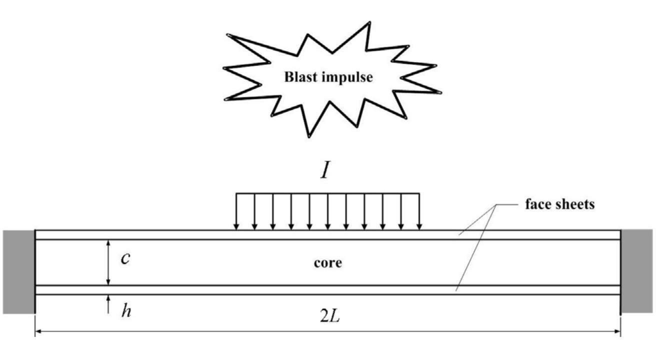

Fig. 1. Sketch of a fully clamped sandwich beam under impulsive loading over a central patch.

2.1. Core compression stage

It is assumed that the impulse I per unit length imparts to the upper face sheet with a velocity $V_0 = I/(\rho_f h)$. According to the momentum conservation, the final common velocity of the face sheets and the core of the central patch with length $2a$ is given by

$$V_f = \frac{I}{2\rho_f h + \rho_c c} \tag{1}$$

Neglecting the rate effect and considering the plastic energy dissipation in compressing the core at a stress σ_n, the average compressive strain ε_c over the entire thickness of the core can be expressed as [15],

$$\varepsilon_c = \frac{\bar{I}^2}{2\bar{\sigma}_n \bar{c}^2 \bar{h}} \cdot \frac{\bar{\rho} + \bar{h}}{\bar{\rho} + 2\bar{h}} \tag{2}$$

where $\bar{I} = I/(L\sqrt{\sigma_f \rho_f})$, $\bar{h} = h/c$, $\bar{c} = c/L$, $\bar{\rho} = \rho_c/\rho_f$ and $\bar{\sigma}_n = \sigma_n/\sigma_f$. However, if ε_c exceeds the densification strain ε_D, ε_c is then set to the densification strain ε_D.

2.2. Bending and stretching stage

At the end of the core compression, the followed stage comprises a combination of the sandwich beam bending and longitudinal stretching. The loaded portion of the sandwich beam has a core thickness $c' = c(1 - \varepsilon_c)$ and a uniformly distributed velocity V_f while the uncompressed portions of the sandwich beam with thickness c keep stationary. For the uncompressed sandwich cross-section with thin, strong face sheets and a thick, weak core, the yield criterion in Ref. (N,M) space [36] is

$$|m| + |n| = 1 \tag{3}$$

where $m = M/M_p$ and $n = N/N_p$ with M_p being the fully plastic bending moment and N_p the fully plastic axial (membrane) force

$$M_p = \sigma_f h(c + h) + \sigma_l \frac{c^2}{4} \tag{4a}$$

and

$$N_p = \sigma_l c + 2\sigma_f h \tag{4b}$$

For the compressed cross-section with an average compressive strain ε_c, it is assumed that the plastic axial force is insensitive to the degree of core compression [15]. The fully plastic axial force for the compressed cross-section is then given by

$$N'_p = N_p = \sigma_l c + 2\sigma_f h \tag{5a}$$

while the fully plastic bending moment is reduced to

$$M'_p = \sigma_f h[c(1 - \varepsilon_c) + h] + \sigma_l \frac{c^2(1 - \varepsilon_c)}{4} \tag{5b}$$

The yield criterion for compressed sandwich cross-section in Ref. (N,M) space is

$$|m'| + |n'| = 1 \tag{6}$$

where $m' = M/M'_p$ and $n' = N/N'_p$ are the non-dimensional bending moment and axial force, respectively.

According to the associated plastic flow rule, the plastic strain rate vector is outwardly perpendicular to the yield curve. That is