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Determination of the wave propagation coefficient of viscoelastic SHPB: Significance for characterization of cellular materials

Hafiz Sana Ullah Butt [a], Pu Xue [a,b,*]

[a] School of Aeronautics, Northwestern Polytechnical University, Xi'an 710072, PR China
[b] State Key Laboratory of Explosion Science and Technology, Beijing, PR China

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Test bars made of viscoelastic materials are frequently employed for the testing of soft materials, using split Hopkinson pressure bar (SHPB) techniques, because of their low mechanical impedance. Determination of the propagation coefficient for such bars is a critical step for the subsequent evaluation of the material properties of the specimen. This propagation coefficient may be determined through experiments or using the analytical solutions if the material properties of the bars are known in advance. Contrary to the case of elastic materials, it is difficult to provide generic properties for such materials as these are dependent on the loading rate, environmental history and manufacturing conditions. Many studies may be found in the open literature reporting numerical values of the identified parameters for various viscoelastic materials evaluated through the wave propagation experiments. However, the observed scatter among such data in the case of individual materials dictates that the published parameters should be used with caution.

Two polymethyl methacrylate (PMMA) bars, used as incident and transmitter bar in an SHPB test setup, are being subjected to the wave propagation testing. Longitudinal strains, generated as a result of axial impact of strikers with two different lengths and recorded at the mid-length of the bars, are used to determine the wave propagation coefficient. Propagation coefficients are also evaluated using selected material models of PMMA published in the literature. A considerable scatter is found in the evaluated frequency dependent propagation coefficient. The consequence of using such scattered properties for the bars on the results of the stress–strain response of aluminum foam is being investigated. Although, the evaluated dynamic properties of the tested foam are not considerably influenced in quantitative terms, however qualitative differences are observed.

1. Introduction

The split Hopkinson pressure bar (SHPB) is one of the best available and most widely accepted techniques for the determination of the stress–strain characteristics of materials at high stain rates. In this technique, the longitudinal impact of a striker bar generates an incident pulse in a first bar (incident bar). This pulse subsequently loads a specimen, dynamically, sandwiched between the first bar and a second bar (transmitter bar). The strains developed due to propagating waves are conventionally recorded at the middle of the bars and are time shifted at the bar–specimen interface for the analysis of the specimen behavior [1,2]. However, this procedure is valid only if the bars are elastic and if the geometric/inertia effects are neglected.

It is well established that the difference between the mechanical impedance of the bars and the tested material should be kept within a reasonably close range to increase the strength of the transmitted signal [3]. For materials having cellular microstructures, e.g. foams and honeycombs, Hopkinson bars also need to be of relatively large diameter to ensure proper representation of the basic material structure of the specimens. Hence, bars with low mechanical impedance, generally made of viscoelastic materials, and of large diameters become the choice in experimental studies of soft materials under high strain rate loading.

The wave propagation in viscoelastic bars is susceptible to both material and geometrical dispersions [2,4]. Due to these effects, the shape of the wave does not remain the same while traveling along a viscoelastic bar. Corrections need to be carried out on recorded strain histories in order to describe those at the bar–specimen interface. To carry out such corrections, the wave propagation

* Corresponding author. School of Aeronautics, Northwestern Polytechnical University, Xi'an 710072, PR China. Tel./fax: +86 29 88494859.
E-mail address: p.xue@nwpu.edu.cn (P. Xue).

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coefficient of the viscoelastic bar or the material properties of the bar material must be known.

Determination of the propagation coefficient for such bars is a critical step for the subsequent evaluation of the material properties of the specimen. The evaluated material properties of the specimen, dynamically tested using viscoelastic SHPB, strongly depend on the accuracy of the propagation coefficient of the both of the bars (incident and transmitter). This propagation coefficient may be determined either using the analytical solutions or through wave propagation experiments. The former method requires that the material properties of the bars be known in advance. While contrary to the case of elastic materials, it is difficult to provide generic properties for such materials as these are dependent on the loading rate, environmental history and manufacturing conditions. The frequency dependent characteristics of the two bars manufactured using the same procedure but with different diameters may be found different e.g. Refs. [11,16]. Such facts necessitate that the published material parameters of viscoelastic materials be used with caution. On the other hand, the direct determination of the wave propagation coefficient through experiments is sensitive to the experimental setup e.g. the geometry of striker used to generate the impact pulse. This study is focused on the variation of the wave propagation coefficient of the bars based on different striker lengths used in the wave propagation experiments, difference of frequency dependent characteristics of incident and transmitter bars, and comparison of experimentally determined wave propagation coefficients with those evaluated through selected material models published in the literature. Furthermore, the effect of the observed variations and scatter on the evaluated response of a low impedance cellular material is explored.

Earlier studies on analytical solutions for the wave propagation in elastic cylindrical rods may be found in e.g. Ref. [5,6,7]. Bancroft [8] solved for the first mode of Pochhammer frequency equation [5] and evaluated the relation between the phase velocity and the wave number. While for the viscoelastic cylindrical rods, Zhao and Gary have formulated the complex frequency equation which can be solved to find out the frequency dependent complex wave number (phase velocity and attenuation coefficient). But such analytical solutions require that the frequency dependent material parameters of the bar material be known in advance, as stated earlier. Alternately, propagation coefficient may also be determined through wave propagation experiments. Lundberg and Blanc [9,10] used the transient pulses and Fourier transform techniques to measure the phase velocity and attenuation. Bacon [11] adopted a fully experimental approach to correct both material dispersion and geometric dispersion in Hopkinson bar experiments.

Furthermore, apart from the correction of dispersion and attenuation of signals, the wave propagation coefficient may also be utilized as the baseline of material identification procedures for viscoelastic materials. Lundberg and Blanc [9,10] evaluated the storage and loss moduli of the polymeric material, from the wave propagation coefficient, by employing 1D wave propagation solutions. Sogabe and Kishida [12] and Sogabe and Tsuzuki [13] employed wave propagation methods in the frequency domain for different viscoelastic material models. Hillstrom et al. [14], S. Mousavi et al. [15] and Mossberg et al. [17] identified parameters of viscoelastic materials by utilizing strains at multiple locations on the impacted bars.

In this study two PMMA bars, used as incident and transmitter bar in an SHPB test setup, are being subjected to the wave propagation testing. Longitudinal strains, generated as a result of axial impact of strikers with two different lengths and recorded at the mid-length of the bars, are used to determine the wave propagation coefficient. Propagation coefficients are also evaluated using selected material models of PMMA reported in the literature. A considerable scatter is found in the evaluated frequency dependent characteristics. The consequence of using such scattered properties for the bars on the results of the stress–strain response of aluminum foam is being investigated.

2. Theory of wave propagation in viscoelastic bar

2.1. Wave propagation in viscoelastic bars neglecting geometric effects

Consider a straight, cylindrical, slender bar made of a linearly viscoelastic material with cross-sectional area and density A and ρ, respectively. It is axially impacted by another bar. If the smallest wavelength of the impact pulse is much greater than the lateral dimensions of the bar, then the lateral motion of the bar can be neglected. The normal stress $\sigma(x,t)$ and the longitudinal strain $\varepsilon(x,t)$ are related to the axial displacement $u(x,t)$ at any cross-section x at time t by

$$\frac{\partial\sigma(x,t)}{\partial x} = -\rho\frac{\partial u^2(x,t)}{\partial t^2} \tag{1}$$

$$\varepsilon(x,t) = \frac{\partial u(x,t)}{\partial x} \tag{2}$$

Using Fourier transform, Eqs. (1) and (2) may lead to one-dimensional equation of axial motion in frequency domain, as follows

$$\frac{\partial^2}{\partial x^2}\hat{\sigma}(x,\omega) = -\rho\omega^2\hat{\varepsilon}(x,\omega) \tag{3}$$

where $\hat{\sigma}(x,\omega)$ and $\hat{\varepsilon}(x,t)$ denote the Fourier transforms of the stress and strain, respectively. ω is the angular frequency in radians/s.

Then, the linear viscoelastic behavior of the material can be expressed as follows:

$$\hat{\sigma}(x,\omega) = E^*(\omega)\hat{\varepsilon}(x,\omega) \tag{4}$$

where $E^*(\omega)$ is the complex Young's modulus of the viscoelastic material. Its real and imaginary parts correspond to storage and loss moduli, E' and E'' respectively, thus $E^*(\omega) = E'(\omega) + iE''(\omega)$, where i is imaginary unit equal to $\sqrt{-1}$.

The frequency dependent propagation coefficient $\gamma(\omega)$ is defined by

$$\gamma^2(\omega) = -\frac{\rho\omega^2}{E^*(\omega)} \tag{5}$$

Moreover, the propagation coefficient $\gamma(\omega)$ is associated to the attenuation coefficient $\alpha(\omega)$, wave number $k(\omega)$ and to the phase velocity $C(\omega)$ by

$$\gamma(\omega) = \alpha(\omega) + ik(\omega) = \alpha(\omega) + i\frac{\omega}{C(\omega)} \tag{6}$$

The attenuation coefficient $\alpha(\omega)$ is representative of damping of the material and is an even function, positive for both positive and negative frequencies, while the wave number $k(\omega)$ represents the wave dispersion and is an odd function, positive only for $\omega > 0$. Both functions are continuous functions. For the case of wave propagation through a viscoelastic bar, the frequency dependent phase velocity may be obtained as $C(\omega) = \omega/k(\omega)$.

Using Eqs. (3)–(5), the one-dimensional equation of axial motion of a viscoelastic bar will become as