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Introducing objective power law rate dependence into a visco-elastic material model of bovine cortical bone

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ABSTRACT

The strain rate dependent behaviour of some visco-elastic materials can be modelled accurately in 1-D by including a stress contribution which depends non-linearly on strain rate. A visco-elastic model by Shim et al. (2005) [3] comprises a Voigt and Maxwell element in parallel and provides an effective representation of cancellous bone from the human cervical spine. The present study demonstrates that the model by Shim et al. is also suitable for modelling the strain rate dependent compression of cortical bone from bovine femurs. Shim et al. found that the model requires a Voigt dash-pot contribution which is proportional to $\dot{\varepsilon}^{1/2}$ in order to model specimen response accurately over a large range of strain rates. Shim et al. proposed an expansion of the 1-D formulation to 3-D where the 1-D strain rate is replaced with a function of the strain rate tensor. This paper provides a frame invariant version of the model by Shim et al. which allows general power law rate dependence for the 3-D case. The response of the model is investigated under a load condition which comprises of an axial deformation and a shear twist. The model is implemented in a commercial finite element package and is used to simulate quasi-static and dynamic bovine bone compression experiments.

1. Introduction

The addition of protective structures to automotive and military vehicles potentially results in the attenuation of the magnitude and rate of occupant loading in the event of vehicular blast or impact. The resulting rate of deformation which is applied to strain rate sensitive occupant body structures (such as bone and soft tissue) could vary unpredictably in such an event. The injury assessment of vehicle occupants in hazardous environments is greatly assisted by computational modelling [1]. The design and evaluation of protective vehicle structures therefore requires constitutive models of bio-material response at a variety of loading rates.

The focus of this study is to formulate and evaluate a constitutive relation that accurately represents the strain rate dependent response of cortical bone over a large range of strain rates. The load bearing ability of long bones is ascribed to cortical bone, which is a dense laminar tissue [2]. Literature concurs that the suitable material models for cortical bone are visco-elastic [3–5]. Mechanical spring-dash-pot models facilitate the conceptual understanding of visco-elastic material response [6]. A linear spring represents a linear increase in material stress, which is proportional to the instantaneous deformation, ε. A dash-pot produces a stress which is proportional to the strain rate, $\dot{\varepsilon}$, at any instant. The Voigt model comprises the parallel arrangement of a single spring and dash-pot, whereas a Maxwell model is the combination of a spring and dash-pot in series [7]. The simple Maxwell and Voigt models are regularly implemented in multiples or combination to model actual materials [3,7].

Tennyson et al. [5] determined the dynamic stress–strain response of bovine cortical bone on a split Hopkinson pressure bar. The rate of deformation ranged between $\dot{\varepsilon} = 10\ \mathrm{s}^{-1}$ to $4.5 \times 10^2\ \mathrm{s}^{-1}$ in the course of these experiments. A Voigt model was used to represent the experimental response of bovine cortical bone:

$$\sigma(t) = k_1 \varepsilon(t) + \eta_1 \dot{\varepsilon}(t). \tag{1}$$

This material description requires the determination of two parameters: an elastic parameter, k_1, and a viscous parameter, η_1. Tennyson et al. [5] determined that the elastic parameter, k_1, reflects the value of the Young's modulus, E.

Tanabe and Kobayashi [4] investigated the dynamic compressive response of bovine cortical bone prior to yield. Experimental results

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indicated that the stress–strain response of bovine cortical bone is linear in quasi-static compression, but clearly non-linear under impact conditions. Tanabe and Kobayashi [4] adopted a non-linear visco-elastic model to represent the mechanical behaviour of bone which comprises a non-linear spring and a non-linear dash-pot in parallel:

$$\sigma(t) = k_1\left(\varepsilon(t) - \alpha\varepsilon(t)^2\right) + \eta_1\left(\frac{\dot{\varepsilon}(t)}{\dot{\varepsilon}_0}\right)^P. \tag{2}$$

here k_1, α, η_1, P and $\dot{\varepsilon}_0$ are material parameters. Again, parameter identification analyses indicated that the elastic parameter value, k_1, assumes the value of the quasi-static Young's modulus. The non-linear elastic term limits the contribution of the spring component as the strain increases.

The approach of non-linear strain rate dependency was also utilised by Shim et al. [3], who implemented a 1-D visco-elastic model that successfully captures the load-bearing response of cancellous bone from the human cervical spine over a large range of strain rates ($\dot{\varepsilon} = 10^{-2}\text{s}^{-1}$ to $1.2 \times 10^3\ \text{s}^{-1}$). The strain rate dependent model comprises a Maxwell and a Voigt element in parallel, as depicted in Fig. 1. The stress response of this model is given by

$$\sigma(t) = k_1\varepsilon(t) + \eta_1\dot{\varepsilon}(t)^{\frac{1}{2}} + \int_0^t k_2\dot{\varepsilon}(\tau)e^{-\frac{t-\tau}{\theta_2}}\mathrm{d}\tau, \tag{3}$$

where k_1, k_2, θ_2 and η_1 are material parameters.

The static response of the model is dominated by the linear elastic term that depends on bone density. A density independent visco-elastic term is added to the elastic term to account for strain rate sensitivity under dynamic loading conditions. This second term incorporates a viscous effect with a non-linear dependence on strain rate. Specifically, Shim at al. used a power law relationship to model the non-linear rate dependence i.e. $\dot{\varepsilon}(t)^{1/2}$.

Shim et al. [3] defined k_1 as the quasi-static Young's modulus

$$k_1 = E_0\rho^\beta, \tag{4}$$

where E_0 and β are material parameters and ρ represents fresh bone density. Fresh bone density is defined by Shim et al. [3] as the mass of a specimen divided by the entire specimen volume before testing, and we also use this definition in this paper. The relaxation time, θ_2, (from Eq. (3)) is defined as the ratio of the Maxwell damping coefficient to the spring coefficient:

$$\theta_2 = \frac{\eta_2}{k_2}. \tag{5}$$

Shim et al. [3] extended the 1-D visco-elastic model to 3-D such that it contained the 1-D formulation as a special case, by replacing the 1-D strain variable ε by an equivalent 3-D strain tensor:

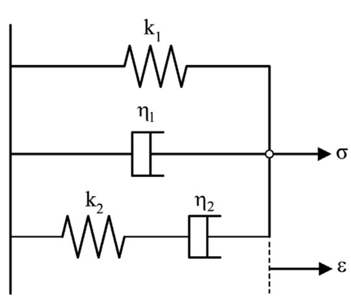

Fig. 1. Diagrammatic representation of the visco-elastic model by Shim et al. [3].

$$\boldsymbol{\varepsilon}_{3\mathrm{D}}(t) = \frac{1}{1+\nu}\left(\frac{\nu}{1-2\nu}\mathrm{tr}(\boldsymbol{\varepsilon}(t))\boldsymbol{I} + \boldsymbol{\varepsilon}(t)\right), \tag{6}$$

where $\boldsymbol{\varepsilon}$ is the usual 3-D infinitesimal strain tensor. The 3-D constitutive visco-elastic material model is then given by

$$\begin{aligned}\boldsymbol{\sigma}(t) &= k_1\boldsymbol{\varepsilon}_{3\mathrm{D}}(t) + \eta_1(\dot{\boldsymbol{\varepsilon}}_{3\mathrm{D}}(t))^{\frac{1}{2}} + \int_0^t k_2\dot{\boldsymbol{\varepsilon}}_{3\mathrm{D}}(\tau)e^{-\frac{t-\tau}{\theta_2}}\mathrm{d}\tau \\ &= \frac{k_1}{1+\nu}\left(\frac{\nu}{1-2\nu}\mathrm{tr}(\boldsymbol{\varepsilon}(t))\boldsymbol{I} + \boldsymbol{\varepsilon}(t)\right) + \frac{\eta_1}{(1+\nu)^{\frac{1}{2}}}\left(\frac{\nu}{1-2\nu}\mathrm{tr}(\dot{\boldsymbol{\varepsilon}}(t))\boldsymbol{I} + \dot{\boldsymbol{\varepsilon}}(t)\right)^{\frac{1}{2}} \\ &\quad + \int_0^t \frac{k_2}{1+\nu}\left(\frac{\nu}{1-2\nu}\mathrm{tr}(\dot{\boldsymbol{\varepsilon}}(\tau))\boldsymbol{I} + \dot{\boldsymbol{\varepsilon}}(\tau)\right)e^{-\frac{t-\tau}{\theta_2}}\mathrm{d}\tau.\end{aligned} \tag{7}$$

Note that the non-linear rate dependence of the Voigt damping term, $\eta\dot{\varepsilon}^{1/2}$, is now extended to the second term in Eq. (7) in 3-D. For the specific relation, developed by Shim et al. [3], this implies that the equivalent 3-D form of $\dot{\varepsilon}^{1/2}$ must be found. However, as is apparent from the relation by Tanabe and Kobayashi [4] (Eq. (2)), it is possible that the viscous strain rate contribution could have an arbitrary power law rate dependence, which will be denoted by P.

The present work presents the strain rate dependent compressive response of bovine cortical bone as determined by experiments. A suitable 3-D visco-elastic model is identified to represent the strain rate dependent compression of bovine cortical bone. The extension of this model to 3-D requires the introduction of objective power law rate dependence as will be shown in subsequent sections. The 3-D model is implemented as a user material in finite element analyses of both a quasi-static and a dynamic bone compression experiment. The performance of the material model is evaluated in a comparison between the experimental and finite element simulation results.

2. Material and methods

2.1. Stress–strain response

The strain rate dependent responses of cortical bone from bovine femurs were determined by quasi-static compression on a conventional testing machine (at strain rates of $10^{-4}\ \text{s}^{-1}$, $10^{-3}\ \text{s}^{-1}$, $10^{-2}\ \text{s}^{-1}$ and $10^{-1}\ \text{s}^{-1}$) and dynamic experiments on a split Hopkinson pressure bar (at strain rates of $2.5 \times 10^2\ \text{s}^{-1}$ and $10^3\ \text{s}^{-1}$). A detailed description of the experimental procedures is given elsewhere [8–10]. Care was taken to keep the strain rate of compression constant in order to prevent the measurement of smeared bone properties [11]. Conical strikers were used to shape the Hopkinson bar input pulse such that the strain rate of compression remained constant in dynamic tests. Fig. 2 shows a sample of the recorded Hopkinson bar stress waves as well as the resulting specimen stress–strain and strain rate responses. The experimental stress–strain curves of all the tested samples are presented in Fig. 3 for the range of strain rates between $10^{-4}\ \text{s}^{-1}$ and $10^3\ \text{s}^{-1}$. It is clear that bone responds to quasi-static and high strain rate compression in two distinct corridors. The dynamic response corridor is significantly more stiff than the quasi-static one.

2.2. Failure and fracture

The damage of bone tissue is likely to be associated with injury in living bone. For the purposes of this study the term *failure stress* indicates the stress at which damage to the bone structure is evident from the stress–strain response. In keeping with the works