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Elastic–plastic response of plates subjected to cleared blast loads

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ABSTRACT

A commonly used approach for the engineering analysis of structures subjected to explosive loads is to approximate the problem as an equivalent Single-Degree-of-Freedom (SDOF) system and to use elastic–plastic response spectra. Currently, the response spectra that exist in the literature do not take into account the fact that blast wave clearing will occur if the target is not part of a reflecting surface that is effectively infinite in lateral extent. In this article, response spectra for equivalent SDOF systems under cleared blast loads are obtained by solving the equation of motion using the linear acceleration explicit dynamics method, with the clearing relief approximated as an acoustic pulse. The charts presented in this article can be used to predict the peak response of finite targets subject to explosions, and are found to be in excellent agreement with a finite element model, indicating that the response spectra can be used with confidence as a first means for predicting the likely damage a target will sustain when subjected to an explosive load. Blast wave clearing generally serves to reduce the peak displacement of the target, however it is shown that neglecting clearing may be unsafe for certain arrangements of target size, mass, stiffness and elastic resistance.

1. Introduction

The intense loading produced from a high explosive detonation can cause significant damage to structural elements, potentially resulting in failure, structural collapse and loss of life. In order to best protect civilian and military infrastructure from explosions, it is important to understand and be able to predict the performance of key components subjected to blast loads.

Numerical analysis methods can be used to model the dynamic response of structures subjected to explosive loads. Finite element (FE) simulations, for example, can model the detonation process, blast wave propagation through air and subsequent interaction with the target [1–4], as well as complex geometries and material nonlinearities [5,6]. Whilst these methods often produce results that are in excellent agreement with experimental observations, high levels of complexity and long analysis times often render such simulations unsuitable, especially during the early stages of design.

Alternative analysis methods may be used, particularly when assessing the approximate level of damage a target will sustain before more refined analyses are undertaken. The Unified Facilities Criteria Design Manual (UFC-3-340-02), *Structures to Resist the Effects of Accidental Explosions* [7], recommends the use of the equivalent Single-Degree-of-Freedom (SDOF) method 8. The SDOF method is often favoured because of its ease of use, relatively few input requirements and available guidance in the literature [7,9,10], and is usually presented as design charts in the form of response spectra.

In these response spectra, the peak dynamic displacement of the target can be obtained from knowledge of the magnitude of the applied load and the ratio of the load duration to response time of the target. These charts were first produced by Biggs [8] and are based on the assumption of a linearly decaying blast load, rather than the exponential 'Friedlander' decay used in the well-established empirical load prediction method of Kingery and Bulmash [11] and ConWep [12]. This limitation has been addressed by Gantes and Pnevmatikos [13], where response spectra are provided for exponential loading, under the assumption that the target is part of a reflecting surface that is infinite in lateral extent.

In the case of reflecting surfaces that cannot be said to be infinite, it is well known that blast wave clearing can significantly reduce the late-time pressure acting on the target face [14–17], reducing the total reflected impulse by up to 50% [18,19]. The influence of clearing on the response of elastic targets subjected to blast loads has recently been investigated by the current authors [20,21], however the effect of target plasticity remains un-quantified. The purpose of this paper is two-

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Nomenclature

A	panel area
b	waveform parameter (decay of exponential pressure time curve)
c	damping coefficient
d	thickness
E	Young's modulus
F	force
F_e	equivalent force
$F_{e,\max}$	peak equivalent force
$F_{e,\min}$	peak negative phase equivalent force
H	scaled target height
i_r	reflected positive phase specific impulse
I	second moment of area
k	stiffness
k_e	equivalent stiffness
K_L	load factor
K_M	mass factor
K_S	spatial load factor
L	span
m	mass
m_e	equivalent mass
M_m	moment capacity at mid-span
p	pressure
$p_{r,\max}$	peak reflected pressure
R	range from charge centre (stand-off),
R_u	elastic resistance
t	time
t_a	time of arrival of blast wave
t_d	positive phase duration
t_d^-	negative phase duration
$t_{d,\text{lin}}$	positive phase duration (linear approximation)
T	natural period
W	explosive mass
x	length along beam
z	displacement
z_E	elastic limit
$z_{\max}$	peak displacement
$z_{\max,\text{inf}}$	peak displacement under exponential (non-cleared) load
$z_{\max,\text{lin}}$	peak displacement under linear load
$\dot{z}$	velocity
$\ddot{z}$	acceleration
Z	scaled distance ($R/W^{1/3}$)
ρ	density
σ_y	yield strength
ϕ	normalised deflected shape

fold: firstly, to develop a complete set of response spectra for finite targets subjected to blast loads, and secondly, to compare these spectra to existing guidance to quantify the effect of blast wave clearing.

2. Elastic–plastic SDOF systems

2.1. The SDOF method

The dynamic equation of motion of a distributed system, for example a simply supported beam with a transiently varying, spatially uniform load (as in Fig. 1) is given as

$$m\ddot{z} + c\dot{z} + kz = F(t), \tag{1}$$

where m, c and k are the mass, damping and stiffness of the system, $\ddot{z}$, $\dot{z}$ and z are the acceleration, velocity and displacement, and $F(t)$ is the externally applied force. The equivalent SDOF method 'transforms' the distributed properties of the real life system into equivalent single-point properties, where the displacement of the single-degree system is equated to the point of maximum

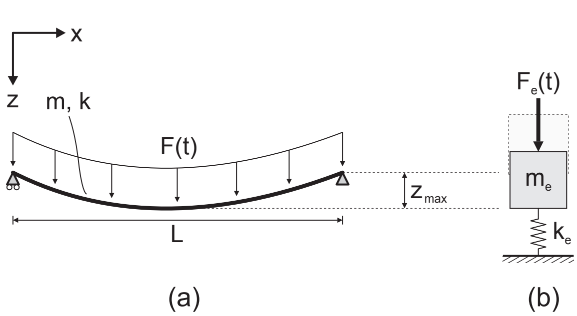

Fig. 1. (a) Distributed and (b) equivalent SDOF systems.

displacement in the distributed system, i.e. displacement at mid-span of a simply supported beam.

Ignoring damping, the dynamic equation of motion of the equivalent system is

$$m_e\ddot{z}(t) + k_e z(t) = F_e(t), \tag{2}$$

where m_e, k_e and $F_e(t)$ are the equivalent mass, stiffness and force. Equating the work done, kinetic energy and internal strain energy of the two systems, the dynamic equation of motion for the SDOF system now becomes

$$K_M m\ddot{z} + K_L kz = K_L F(t). \tag{3}$$

where the mass factor, K_M, and load factor, K_L, are used to transform the distributed properties into the single point equivalent values. The transformation factors for various support conditions and loading distributions, based on the assumption of the normalised deflected shape, ϕ, can be found in the literature [7–10].

2.2. Elastic–plastic response spectra

In the analysis performed by Biggs [8], elastic–plastic SDOF systems are subjected to a linearly decaying uniform load

$$F_e(t) = \begin{cases} F_{e,\max}\left(1 - \frac{t}{t_{d,\text{lin}}}\right), & t \le t_{d,\text{lin}} \\ 0, & t > t_{d,\text{lin}} \end{cases} \tag{4}$$

where $p(x,y,t)$ is the peak force and K_S is the duration of the triangular load. The SDOF system has a bilinear elastic-perfectly plastic resistance function as shown in Fig. 2. This comprises linear elastic behaviour with spring resistance $k_e z$, until the elastic limit, z_E, is reached, followed by plastic behaviour with constant spring resistance, R_u, thereafter. After the peak displacement, $z_{\max}$, is reached, the displacement decreases and the system begins to rebound. When rebounding, the system again behaves elastically until a