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Probabilistic predicting the fatigue crack growth under variable amplitude loading

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ABSTRACT

The probabilistic method presented in this paper facilitates a simplified description of fatigue crack growth under variable amplitude loading and the estimation of fatigue life. Fundamental to the description is a finite difference equation with the coefficients originated from the Paris formula which models the dynamics of crack growth. The characteristic features of crack growth under overload–underload cycles existed in an exploitive loading were modeled by using the modified Willenborg retardation model. The presented probabilistic method has a good confirmation by experimental research of crack behavior and fatigue life estimation for an aeronautical aluminum alloy sheet 2024-T3 subjected to variable amplitude load program. This method needs an extension over the crack initiation period.

1. Introduction

The operational spectrum of a structure is a typical variable amplitude spectrum. Exploitive loading induces in the materials physical phenomena that influence on crack growth behavior. This is called as the effect of load interaction which means the importance both of initial crack length at a given moment and the load time history for the crack growth in the materials. There exist numerous physical mechanisms that accompany the crack extension under single or multiple overloads and underloads imposed cyclically or randomly in the base line load. The most frequently mentioned mechanisms are either plastically induced crack closure and crack rate retardation associated with the plastic zone ahead of a crack tip that it was induced by an tensile overload cycle. Compressive underload cycle, on the other hand, leads to the crack tip sharpening and the crack rate increase. Distribution of residual stresses in the plastic zone, the thickness of a particular component as well as mechanical properties of the material are determining factors that contribute to irregular fatigue crack growth [4–7,17]. Therefore, it is a considerable interest to quantitatively predict the experimental tendency in crack growth behavior due to changes in load, material and geometry of a component. For this goal a certain empirical prediction models (Elber, Wheeler, Willenborg) as well as numerical simulations of crack growth under variable amplitude load derived by the codes FASTRAN, NASGRO, CORPUS and AFGROW find the application.

For predicting fatigue crack growth rate in a component subjected to random loading probabilistic models are proposed. The reasons for applying probabilistic approach are as follow: inhomogeneity of real material, scatter of mechanical properties of the material, randomness of cracking process and technological conditions (quality of manufacturing). There, it is required that the model and the real object are physically identical as regards the time and point of crack initiation, crack propagation period and fatigue lifetime of a component.

The factors mentioned above have an uncertain influence on fatigue crack growth process and lead to the scatter both of critical crack size and fatigue lifetime being experimentally estimated. This problem is particularly important in the case of damage tolerance design at random loading. To describe this influence the fundamental parameters were randomized and transformed into statistical distribution function.

Exemplary reviews of existing probabilistic models were derived earlier by Castiglioni [8] and later by Kim and Shim [9] and Castillo et al. [10] as well. Generally, in the literature stochastic crack growth models are based upon two approaches. One type of them derives models from randomization of deterministic equation of crack growth by providing the distribution of the random time to crack length and non-negative random variable $X(t)$: $da/dN = f(\Delta K)X(t)$, while $X(t)$ is a Gaussian random noise. A special case of time random variable to reach a critical crack size was proposed by Tang and Spencer [11] modeling the Virkler data by two-state Markov process. The tests were conducted for assumed certain distribution functions such as lognormal, normal and Weibull. A stochastic Markov chain model based on the Paris–Erdogan equation was applied to describe the distribution of fatigue life under constant stress intensity factor range in [9]. According to the authors the method mentioned above requires a small number of tests to describe the variability of fatigue crack growth.

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For modeling random fatigue crack growth a model based on cumulative random jump process was proposed by Sobczyk and Trębicki [12]. In the model, fatigue process was characterized by dominant crack length and represented by a random sum of random elementary increments of Poison distribution function. For prediction of fatigue crack growth Ghonem and Provan [13] adopted discontinuous Markov random process. A general methodology for stochastic fatigue life prediction under variable loadings was proposed in [14]. To achieve this goal a nonlinear fatigue damage accumulation rule and a stochastic S–N curve were combined. A new probabilistic model was considered for fatigue crack growth prediction which was based on the Wöhler curve experimentally determined for actual component [10]. Extreme value theory was applied to derive the density of crack size.

The second type of the stochastic approaches derives models from randomization of the Paris–Erdogan equation. In these models a generalized the Fokker–Planck equation describes the temporal variability of crack length distribution. Using the solution of this equation the fatigue life distribution can be determined.

Early fatigue crack growth stage was modeled based on continuum damage mechanics applied to each grain and grain boundary [15]. The properties of the grains to damage accumulation were considered to be random variables described by the Weibull distribution. Probabilistic approach was applied to predict short and long crack growth regimes using two-parameter Weibull distribution for expressing the resistance of grain boundaries to the fatigue [16]. Fundamental to the description were finite differences equation as well as the Fokker–Planck differential equation which modeled the dynamics of crack growth.

In the present paper a probabilistic approach to predicting fatigue crack growth rate under variable amplitude loading with imposing multiple overload–underload cycles is developed on the basis of modified Willenborg model [2]. The capacity of this model will be experimentally verified for the 2024-T3 Alclad aluminum alloy sheets subjected to variable amplitude loading. The influence of the shape of loading spectrum on crack rate is analyzed by means of electron microscopes SEM and TEM. The details of the microfractographic analysis concerning the 2024-T3 alloy and its correlation with the fatigue crack growth rate under single and multiple overloads–underloads can be found in [1].

2. Probabilistic method of fatigue crack growth rate predicting

In order to calculate the retardation effect on the crack rate due to overload–underload cycles the improved Wheeler model named as the Willenborg model was applied. Both these models are based on the assumption that crack growth is controlled not only by the plastic zone but also by residual deformation left in the wake of the crack as it grows through previously deformed material [2]. In the Willenborg model there was introduced a reduced stress σ_{red} which is needed to get through the plastic zone $r_{p.OL}$ created by the tensile-overload cycle. In the case of an underload half-cycle with compressive stress either the current plastic zone of the radius r_{pi} or the overload plastic zone of the radius $r_{p.OL}$ ahead of the crack tip are reduced by the radius $r_{cp.}$ In accordance with [2] the plastic zone radius r_{cp} which results from the interaction of an elastic material in the vicinity of growing crack is determined by:

$$r_{cp} = \frac{1}{D\cdot\pi}\left(\frac{\Delta K_{UL}}{2\cdot\sigma_{0.2}}\right)^2 = \frac{1}{D\cdot\pi}\left(\frac{K^* - K_{min,UL}}{2\cdot\sigma_{0.2}}\right)^2 \quad (1)$$

where $D = 2$ (plane stress state) or 6 (plane strain state), $K^* = \min\left(K_{min}^{CA}, K_{th}\right)$; K_{min}^{CA} is the minimum stress intensity factor in a base CA cycle; K_{th} is the threshold stress intensity factor; $K_{min,UL}$ is the minimum K associated with the underload (UL); $\sigma_{0.2}$ is the offset yield stress.

The retardation factor C_p is given by [2]:

$$C_p = \left(\frac{r_{pi} - r_{cpi}}{a_{OL} + (r_{p.OL} - r_{cp.UL}) - a_i}\right)^n \quad (2)$$

where a_i is the current crack length corresponding to the ith cycle; a_{OL} the crack length at which overload is applied; r_{pi} the current plastic zone size corresponding to the ith cycle; r_{cpi} the current compressive plastic zone size corresponding to the ith cycle; $r_{p.OL}$ the plastic zone size due to tensile overload; $r_{cp.UL}$ the compressive plastic zone size due to compressive underload and n is the shaping exponent, which is generally obtained through experiments.

The stress redistribution occurs ahead of the crack tip as a result of a tensile reaction of an elastic material surrounding the growing crack and the compressive stresses acted in the monotonic plastic zone ahead of this crack. Assuming that the reduced stresses σ_{red} operate in the plastic zone, the condition for the crack growth retardation in the Willenborg model is as follows [2]:

$$\begin{aligned} a_{OL} + (r_{p.OL} - r_{cp.UL}) &= a_i + \frac{1}{D\cdot\pi}\left(\frac{K_{red}}{\sigma_{0.2}}\right)^2 \\ &= a_i + \frac{1}{D\cdot\pi}\left(\frac{\sigma_{red}\cdot\sqrt{\pi\cdot a_i}M_k}{\sigma_{0.2}}\right)^2 \end{aligned} \quad (3)$$

hence, the stress required for getting through the plastic zone is determined by the equation:

$$\sigma_{red} = \frac{\sqrt{2}\cdot\sigma_{0.2}}{M_k}\cdot\sqrt{\frac{a_{OL} + (r_{cp.OL} - r_{cp.UL}) - a_i}{a_i}} \quad (4)$$

where M_k is the geometrical factor; $\sigma_{0.2}$ the offset yield stress; a_i the current crack length corresponding to the ith cycle; a_{OL} the crack length at which overload is applied; $r_{p.OL}$ the plastic zone size due to tensile overload and $r_{cp.UL}$ is the compressive plastic zone size due to compressive underload.

In the model, it is assumed that the value of compressive stresses σ_c existed in the overload plastic zone is equal to the reduced stress σ_{red} minus the maximum applied overload stress, that is $\sigma_c = \sigma_{red} - \sigma_{max,OL}$. The values of $\sigma_{max,i}$ and $\sigma_{min,i}$ are reduced by the compressive stress σ_c in each load cycle. In a fatigue cycle the values of effective maximum $\sigma_{max\ eff,j}$ and minimum stresses $\sigma_{min\ eff,j}$ equal respectively to:

$$\begin{aligned} &\sigma_{\max eff,j} = \sigma_{\max,j} - \sigma_c = 2\sigma_{\max\ ,j} - \sigma_{red} \\ &\sigma_{min\ eff,j} = \sigma_{min,j} - \sigma_c = \sigma_{min,j} + \sigma_{\max,j} - \sigma_{red} \\ &\max(0, \sigma_{eff,j}) = \begin{cases} 0 & \text{where } 0 \geqslant \sigma_{eff,j} \\ \sigma_{eff,j} & \text{where } \sigma_{eff,j} > 0 \end{cases} \end{aligned} \quad (5)$$

The effect of the stress ratio R on crack growth rate should be taken into account while calculating effective stress changes are replaced by a constant-amplitude cyclic zero-to-tension load program ($R = 0$) according to:

$$\Delta\sigma_{eff,j} = \sigma_{max\,eff,j}\cdot(1 - R_j)^r \qquad j = 1\ldots q, \quad R_j \geqslant 0 \quad (6)$$

where $\sigma_{max\ eff,j}$ and R_j are the maximum effective stress and the stress ratio for jth stress block, respectively. The value of the stress modification factor γ is 0.68 for aluminum alloys under variable-amplitude loading with the stress ratio $R \geqslant 0$.

Let us assume that the crack growth rate follows the Paris formula under each stress cycle:

$$\Delta K_j = M_{k,j}\cdot\Delta\sigma_{eff,j}\cdot\sqrt{\pi\cdot a_j} \quad (7)$$

where ΔK_j is the stress intensity factor for the length a_j of the crack; $M_{k,j}$ is the geometrical factor M_k for the length a_j of the crack; and