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Technical note

Fictitious notch rounding concept applied to V-notches with end holes under mode 3 loading

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ABSTRACT

The present technical note is aimed to provide a closed form expression for the microstructural support factor and for the fictitious notch radius in plates weakened by V-notches with root end-holes. Taking advantage of some recent closed form expressions for the stress distributions due to V-notches with end holes the fictitious notch rounding approach is applied here to mode 3 loading. The factor s for the V-notch with end holes is found to be strongly influenced by the opening angle and the new values are compared with the previous solution available in the literature and dealing with blunt V-notches. To validate the new expressions a comparison is carried out between the theoretical stress concentration factor (SCF) obtained from a rounded V-notch with a fictitiously enlarged end hole (of radius ρ_f) and the effective stress concentration factor obtained by integrating the relevant stress over the microstructural characteristic length (MCL), ρ^*, in a pointed V-notch. A sound agreement is found from the comparison. The range of validity of the present equations are limited to linear elasticity or in those cases where the plastic zone is very small with respect to the MCL of the material.

1. Introduction

The concept of "elementary" volume and "Micro Structural Length" was introduced many years ago by Neuber [1–3], who formulated the idea that in the presence of a sharp notch the material is sensitive to an effective stress lower than the real notch stress, while the effective stress results for a fictitious root radius ρ_f.

Starting from the idea that an 'elementary particle' [1] at the notch tip is decisive for the strength of the notched components, a material-dependent microstructural length parameter is defined which is named '(substitute) microstructural length' [1–3]. The concept of microstructural length applied to high-cycle fatigue loading considers the fact that, in the case of sharp notches, the notch stress averaged over a short distance, normal to the notch contour where the fatigue cracks initiate, is the fatigue-effective parameter, instead of the theoretical maximum notch stress.

The radius ρ_f was equal to $\rho + s\rho$ [1], being ρ the actual notch radius ($\rho = 0$ for a pointed V-notch), ρ the 'micro structural support-length' and s a factor accounting for the state of multiaxiality, determined by Neuber for different strength criteria using the stress fields of parabolic notches [3]. The FNR method emphasises that in the presence of a pointed or a sharp notch the theoretical maximum notch stress can no longer be thought of as the static or fatigue strength-effective parameter to be used. In such cases one should use, instead, the notch stress averaged over a short distance normal to the edge. Under high cycle fatigue, the integration path should fairly well coincide with the early crack propagation path. Thanks to the FNR approach, notch stress averaging can be avoided and the effective notch stress to be used for failure assessments can be determined directly by evaluating the maximum stress at a fictitiously enlarged notch tip of radius ρ_f. The hypothesis of microstructural support applied to static loading is relating to brittle fracture and was applied first by Wieghardt [4] and Weiss [5]. The main hypotheses of the FNR approach are that the material obeys a linear elastic behaviour and the plastic zone near the notch tip is small with respect to the characteristic length of the material. The yield threshold can be used to define the range of validity of the approach. These hypotheses are usually verified at high cycle fatigue by structural materials and also for static failure of brittle or quasi-brittle materials.

Following the concepts developed by Neuber [1–3], Radaj first proposed to assess the high-cycle fatigue strength of welded joints (toe and root failures) made of structural steel by inserting a fictitious end hole at the weld toe or root [6–8]. The FNR approach, later extended also to welded joints made of aluminium alloys [9] and magnesium alloys [10], has been introduced in the recent IIW recommendations [11] as well as in the FKM guidelines [12]. A constant end hole radius $\rho_f = 1$ mm, independent of the actual notch opening angle is suggested in the above mentioned standards as also summarised in Ref. [13].

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Nomenclature

a	notch depth
A	characteristic parameter in stress equations for V-notches with root holes
C	characteristic parameter in Neuber's stress equations for V-notches
G	elastic shear modulus
K_3	generalised stress intensity factor for in-plane shear loading (mode 3)
K_t	theoretical stress concentration factor
$K_t(\rho_f)$	stress concentration factor evaluated with fictitious notch rounding
$\overline{K}_t$	stress concentration factor from averaged notch stresses
q	parameter dependent on the notch opening angle in the stress field equations
r, θ	polar coordinates
s	factor quantifying multiaxiality effects
x_0	distance between notch root and origin of the cartesian coordinate system
u, v	curvilinear coordinate in Neuber's stress representation
2α	opening angle of V-notch
Δ	relative deviation
λ_3	Williams' eigenvalue for mode III stress distribution at V-notches
ν	Poisson's ratio
ρ^*	real notch radius
ρ	microstructural support length
ρ_f	fictitious notch radius
τ_{max}	maximum notch stress
τ_{th}	theoretical (equivalent) notch stress
$\tau_{\theta r}$, $\tau_{z\theta}$	stresses in the polar coordinate system
$\bar{\tau}$	notch shear stress averaged over ρ^*
τ_{max}	maximum notch shear stress
τ_n	remote nominal shear stress
τ_{ng}	nominal shear stress in gross section
FNR	fictitious notch rounding
MSED	minimum strain energy density
MCL	microstructural characteristic length
MTS	maximum tangential stress
SCF	stress concentration factor

The FNR approach has been also extended to multiaxial loading by Sonsino and Lagoda considering different values of the fictitious notch radius under mode 1 and mode 3 loading respectively [14]. By a fictitious radius covering the worst case $\rho = 0$ meaning the crack case, fatigue notch factors for bending K_{fb} and for torsion K_{ft} have been evaluated by using different fictitious notch radii for bending ($r_{fb} = 1.16$ mm) and torsion ($r_{ft} = 0.4$ mm).

In a recent paper by Susmel et al. [15] the accuracy of the Modified Wöhler Curve Method (MWCM) [16] in estimating fatigue lifetime of welded joints subjected to multiaxial fatigue loading is investigated when this bi-parametrical critical plane approach is applied in conjunction with the reference radius concept. The local liner-elastic stress fields in the vicinity of weld toes are determined by introducing a reference radius, r_{ref}, having length equal to 1 mm, while the degree of multiaxiality and non-proportionality of the stress fields determined is directly taken into account through the MWCM itself.

In recent years, the small-size notch approach has found application, especially to thin-sheet lap joints ($t = 0.7$–3.0 mm), either resistance spot-welded or laser-beam seam-welded [17,18]. A small-size keyhole or U-shaped notch with a substitute radius ρ_s in the grain size range, $\rho_s = 0.05$ mm, is introduced when modelling the slit tip at the weld root or nugget edge. The maximum elastic notch stress is evaluated and used to characterise endurable stress ranges dependent on number of load cycles in the medium-cycle and high-cycle fatigue range.

Some recent developments [19,20], dealing with blunt V-notches under mode I and mode III loading, have demonstrated that the multiaxiality factor s is very sensitive to the notch opening angle, the assumption of a constant value of the fictitious radius being strictly conservative. A very good correspondence was found between the theoretical stress concentration factors evaluated at the fictitiously rounded notches and the effective stress concentration factors obtained by integrating the relevant stress on the pointed V-notch bisector. Under mode II loading, the problem is more complex than under mode I and mode III, mainly because the maximum elastic stress is outside the notch bisector line [21]. A numerical application of FNR approach has been recently provided [22]. A comparison between the FNR approach and the strain energy density (SED) approach was carried out in Ref. [23] while the basis and the numerical advantages of SED are described in Refs. [24,25].

Dealing with out-of-plane shear loading, which is the topic of the present technical note some basic solutions for the FNR approach were provided by Radaj and Zhang by considering the out-of-plane shear-loaded crack (mode III) and the corresponding fictitious elliptical notch in comparison [26]. Another comparison value of s was obtained from the solution for the out-of-plane shear-loaded keyhole notch [8]. For out-of-plane shear loading of parabolic notches, Neuber [3] gives $s = 1.0$ contrary to Neuber [2] stating $s = 0.5$. On the other hand, $s = 0.5$ is derived by Radaj and Zhang [27] for out-of-plane shear loading of crack tips. As stated above the problem of out-of-plane loading of fictitiously rounded blunt V-notches was discussed in Ref. [20] showing the not negligible influence of the notch opening angle and providing prove of the soundness of the found values of s by means of numerical analyses. For the parabolic notch the value $s = 1.0$ was found suitable being the larger deviation between the theoretical stress concentration factor of the fictitiously rounded notch and the effective, averaged stress concentration factor lower than 4% [20].

Different from Ref. [20] the present work is focused on V-notches with end holes, being this geometry exactly that suggested for the application of FNR approach to welded joints. A recent solution for the mode 3 stress distributions [28] is used with the aim to investigate the influence of the opening angle on the microstructural support factor, s, and, then, on the fictitiously enlarged notch radius, ρ_f. Dealing with V-notches with end holes the analytical frame reported in [28] has been also used to apply the FNR approach to the case of mode 1 and mode 3 loadings [29,30] being the mode 3 the only missing case.

The main outlines of the present technical note are as follows:

– The basis of the FNR approach is first presented.
– Dealing with mode 3 loading and FNR approach the available solutions are reviewed.
– The FNR approach is applied to V-notches with end root holes providing the final expression for ρ_f.
– The new expressions are validated by means of finite element analyses.