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On modeling of fractured media using an enhanced embedded discontinuity approach

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ABSTRACT

The main focus of this study is on application of the enhanced embedded discontinuity approach to the analysis of pre-existing fractures. The J-integral is used to evaluate the energy release rate around the crack tip and its value is compared with both that obtained from Extended FEM simulations as well as from an analytical solution. The approach is also used for modeling of cohesive crack propagation. It is demonstrated that the framework gives results that are very close to those obtained using Extended FEM, while the former requires less computational effort. A comparison with a standard smeared approach is provided in order to highlight the nature of the contribution. The embedded discontinuity framework is also applied to flow problems with pre-existing cracks. A modified form of Fourier law is introduced and later employed for modeling of heat transfer/flow in the domain that contains thermally isolated/impervious cracks.

1. Introduction

This study is an extension of earlier work reported in [1]. The main objective here is to investigate the accuracy (in relation to XFEM) of the approach incorporating an enhanced constitutive law with embedded discontinuity for the analysis of crack propagation. In addition, using a similar methodology, a new modified form of Fourier law is introduced that is capable of describing flow within the domain containing thermally/hydraulically isolated inclusions. A number of numerical examples are provided addressing both of the above issues.

Modeling of damage initiation and propagation has been one the most intensely researched topics over the last few decades. The existing analytical solutions are restricted to an elastic material and involve simple geometries and boundary conditions. Therefore, they are not directly relevant to practical engineering problems. The latter require, in general, a numerical simulation that typically involves the use of the Finite Element Method (FEM). The early methodologies for capturing the progressive damage were based on tracing the crack propagation on the element boundaries [2]. These were later combined with remeshing techniques [3] to increase the accuracy of the simulated crack pattern. In addition, some smeared techniques have also been proposed that incorporated plasticity-based strain-softening relations [4,5]. A detailed review of early FEM-based methodologies for crack propagation analysis can be found in [6]. The concept of enhancing the standard strain-softening frameworks to ensure the mesh-independency was first presented in Ref. [7]. The approach advocates the use of volume averaging to estimate the properties of an initially homogeneous medium intercepted by a shear band/interface. The proposed constitutive relation incorporates the properties of constituents (viz. intact material and interface) as well as a characteristic dimension associated with the structural arrangement, which provides a length scale in the constitutive model. The issue of analysis of strong discontinuities induced by strain softening was later addressed in [8]. In that study,

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a description of discontinuous motion was introduced, via an enriched strain field, and was incorporated into the finite element interpolants [9]. After introduction of the partition of unity approach [10], the Extended Finite Element Method (XFEM) was proposed [11,12], which allows not only for an accurate description of a discontinuous velocity/displacement field but also for incorporation of the tip enrichments into the approximation space. Although some conceptual similarities can be found between the strong discontinuity approach introduced in [8] and the XFEM, the latter provides a more elaborate way of incorporating any kind of asymptotic function into the FEM approximation space. The XFEM approach has been applied to various problems such as cohesive crack problem [13], frictional contact [14], strain localization [15,16], interface and fracture propagation in multi-phase media [17–19]. A detailed literature review can be found in [20]. The primary difficulty in implementing XFEM is the need to deal with additional degrees of freedom. In particular, a special treatment for activating enriched DOFs is required that generally increases the computational effort compared with the standard FEM. The main advantage of this method is the ability to incorporate any asymptotic function in the discretized model.

There are two key topics that can be addressed within XFEM, i.e. modeling of discontinuous motion by enriching the FE interpolants, and a discrete representation of crack. The latter results in a stable approach for advancing the analysis past the ultimate load that triggers the failure. This is in contrast with smeared crack models in which crack can initiate during equilibrium iterations and may form in the adjacent integration points. In discrete representation, the crack direction is fixed within an element and is identified after equilibrium iterations. In this way, the crack formation in two adjacent elements is usually avoided. The crack geometry is defined by the level-set method [21], which provides a smooth propagation path. It is believed here that a discrete representation of crack can result in a more stable algorithm for most smeared damage/crack models. In the recent study by authors [1], a discrete tracing of crack path was used as the key idea along with embedded discontinuity model [7,22] for modeling of both the cohesive crack propagation and strain localization. It was shown that the discrete representation improves the stability of the solution and results in similar global and local stress/strain distribution. A new procedure for deriving the governing equations was provided based on the general description of discontinuous motion. The approach was also used for the modeling of failure in anisotropic media [23]. The methodology employs some concepts originating from XFEM, such as discrete crack propagation strategies and level-set method; in terms of formulation, however, it is based on an enhanced plasticity/damage framework. As a result, a special treatment of elements that involves activating enrichments/distinct integration scheme for XFEM or static condensation in strong discontinuity approach, is no longer required.

The present paper is organized as follows. In the next section, the formulation for both XFEM as well as the standard FEM incorporating a constitutive law with embedded discontinuity (FEM/CLED) is outlined. The section is concluded by presenting an extension of CLED methodology to modeling of discontinuities in scalar field problems. This involves a modified version of Fourier law that can be employed for the analysis of heat transfer/fluid flow problems within domains containing discrete fractures. In Section 3, the procedure for tracing the propagation of crack path is addressed. In Section 4, the numerical simulations of traction free crack are provided, which include the evaluation of J-integral [24]. It is shown that both FEM/CLED and XFEM with Heaviside enrichment, i.e. no singular tip enrichment, result in almost identical energy release rates and stress/strain distribution. In Section 5, the results of numerical analysis of some selected problem involving cohesive crack propagation are addressed and the predictions are compared with experimental benchmarks. The study closes with an analysis involving a heat transfer within a domain containing a set of thermally non-conductive cracks of a random orientation. The results of both mechanical and flow simulations presented here clearly demonstrate that the FEM/CLED framework can be applied to a broad range of problems involving the presence of discontinuities.

2. Description of a discontinuous motion

Following Ref. [1], a discontinuous motion $\mathbf{v}(\mathbf{x}, t)$ in the domain Ω that contains a discontinuity surface Γ_d can be defined as

$$\mathbf{v}(\mathbf{x}, t) = \hat{\mathbf{v}}(\mathbf{x}, t) + \mathcal{H}_{\Gamma_d}\tilde{\mathbf{v}}(\mathbf{x}, t) \tag{1}$$

where, $\hat{\mathbf{v}}(\mathbf{x}, t)$ and $\tilde{\mathbf{v}}(\mathbf{x}, t)$ are continuous functions in the solution domain Ω and $\mathcal{H}_{\Gamma_d} = \mathcal{H}(\phi)$ is the Heaviside step function that can be expressed in its symmetric form as

$$\mathcal{H}(\phi) = 2\int_{-\infty}^{\phi} \delta(\varphi)\, d\varphi - 1 = \begin{cases} +1 & \phi > 0 \\ -1 & \phi \le 0. \end{cases} \tag{2}$$

Here, $\phi = \phi(\mathbf{x})$ is the signed distance from the discontinuity interface Γ_d, and $\delta(\varphi)$ is the Dirac delta function which is defined as being singular at $\phi = 0$ and equal to zero elsewhere. Denoting jump of a function on the discontinuity interface by $[\![\bullet]\!] = \bullet^+ - \bullet^-$, the rate of separation between the opposite crack faces, i.e. $\dot{\mathbf{g}}$, can be defined as

$$\dot{\mathbf{g}} = [\![\mathbf{v}]\!] = \left(\hat{\mathbf{v}} + \mathcal{H}_{\Gamma_d}\tilde{\mathbf{v}}\right)^+ - \left(\hat{\mathbf{v}} + \mathcal{H}_{\Gamma_d}\tilde{\mathbf{v}}\right)^- = \hbar\tilde{\mathbf{v}}$$
$$\hbar = [\![\mathcal{H}]\!] = \mathcal{H}^+ - \mathcal{H}^- \tag{3}$$

where, based on the representation (2), the jump in the step function is $\hbar = 2$. Considering that $\nabla\mathcal{H}(\phi) = \mathcal{H}'\,\nabla\phi$ and $\mathcal{H}'(\phi) = \hbar\delta(\phi) = \hbar\delta_{\Gamma_d}$, the velocity gradient of the discontinuous motion (1) can be expressed as

$$\nabla^s\mathbf{v} = \nabla^s\hat{\mathbf{v}} + \mathcal{H}_{\Gamma_d}\nabla^s\tilde{\mathbf{v}} + \delta_{\Gamma_d}(\hbar\tilde{\mathbf{v}} \otimes \mathbf{n})^s \tag{4}$$

where $\mathbf{n} = \nabla\phi$ is the normal to the interface, and the superscript s refers to the symmetric part of the gradient operator.

2.1. Space discretization for XFEM strategy

Within the XFEM strategy, a discontinuous field can be incorporated into the approximation space by introducing enrichment functions and additional degrees of freedoms. Thus, the discontinuous motion (1) can be approximated