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How does surface tension affect energy release rate of cracks loaded in Mode I?

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ABSTRACT

The role of surface stress on fracture of elastic solids has been studied by two group of researchers with different conclusions. One group concludes that surface stress has no effect on energy release rate except that the singularity of the crack tip field is stronger than the usual inverse square root singularity dictated by linear elastic fracture mechanics. The other group concludes that the singularity of the stress field reduces to a logarithmic singularity, thus implying that the local energy release rate is zero. In this letter we resolve this paradox by examining the solution of a special case where surface stress is isotropic and independent of surface strain. We show that surface tension resists crack growth by lowering the applied energy release rate while retaining the inversely square root singularity of the elastic crack tip field. We demonstrate this idea by solving a perturbation problem where the capillary length is much smaller than the crack length. A closed form expression for the local energy release rate is obtained in the limit of small surface tension.
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1. Introduction

The characteristic length scale that controls the deformation of a solid due to its surface tension σ is given by the ratio of the surface tension to its shear modulus G. For hard materials such as metals and ceramics, this "capillary" length σ/G is smaller than atomic dimensions, hence surface tension effect can be ignored. However, the capillary length of soft materials such as hydrogels and elastomers range from tens of nm to hundreds of μm. For this class of materials, surface tension can drive shape change, for example, a sharp corner in a soft material cannot remain sharp because of surface tension [1,2]. Surface tension can also flatten surfaces of structures made by replica molding [3,4] and cause Plateau instability in thin gel filaments [5]. However, surface tension can resist deformation, for example, both experiments and theory has shown that the contact mechanics of spheres and cylinders on soft elastic substrates can be affected by solid surface tension [6–11]. Liu et al. [12] have shown that surface tension induced Laplace pressure can cause closure of a crack inflated by hydrostatic pressure.

There are very few experimental studies examining the role of surface stress in fracture since actual measurements of surface stresses are very difficult to make. The first theoretical study on the influence of surface stress on the fracture of elastic solids were carried out by Thomson, Chuang and Lin [13] in 1986 (hence forth referred as TCL). In their model, they ignored the curvature of the deformed crack faces, and the effect of surface stresses is modeled as a line force acting at the crack tip. Their analysis showed that even though the singularities due to the line force are much higher than the typical inverse square root

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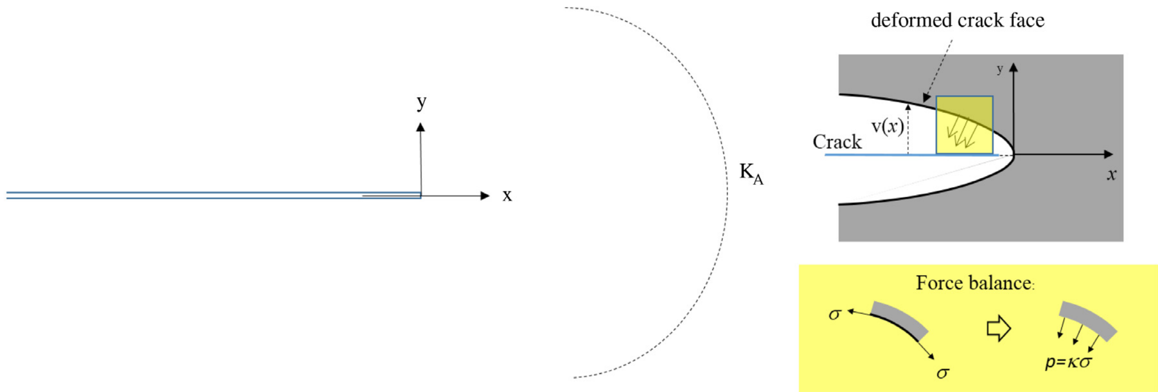

Fig. 1. Figure on left is a schematic of the geometry, where an undeformed semi-infinite plane strain Mode I crack occupying the negative x axis is subjected to a remote far field controlled by the applied stress intensity factor K_A. Figure on right shows the deformed crack face, which is subjected to a Laplace pressure induced by local curvature of the deformed crack surface.

singularity in linear elastic fracture mechanics, they have no effect on the crack tip energy release rate [see remarks after Eq. (42) in their paper].

It is interesting to compare TCL's result with the more recent studies of Kim, Schiavone and Ru [14,15] (hence forth refer as KSR). Using the surface constitutive model of Gurtin and Murdoch [16], KSR [14] in 2010 formulated a small strain theory of fracture to study the effect of surface tension on fracture of a plane strain crack loaded in Mode I and Mode II. Their numerical results showed that the stress field directly ahead of a Mode I crack is bounded, *irrespective of the magnitude of the surface stress* (as long as it is not exactly zero). In a later paper [14], KSR reexamined their numerical solution in greater details and concluded that the stress field directly ahead of the crack tip has a weaker logarithmic singularity. In both cases their numerical results imply that the stress intensity factor (or the local stress intensity factor $K_{\text{local}} = K_A + K_{ST}$) is *exactly* zero. In other words, the applied stress intensity factor K_A is canceled by the negative stress intensity factor K_{ST} caused by the curvature induced Laplace pressure acting to close the crack faces. Since a zero local stress intensity factor implies that local energy release rate is also zero, their result is the exact opposite of TCL's. Furthermore, because surface tension is represented as a line force, the stress field near the crack tip in TCL model is more singular than the inverse square root singularity of the classical theory and this result also contradicts the result of KSR.

The goal of this letter is to resolve this paradox. To simplify the analysis, we focus on Mode I cracks and the surface stress $\sigma_{\alpha\beta}$ is assumed to be isotropic and is a constant independent of surface strain, that is, $\boldsymbol{\sigma} = \sigma \boldsymbol{I}$ where σ is the surface tension (force/length). To focus attention on the crack tip, we consider the "Small Scale Surface tension" problem in which the effect of surface tension is confined to a region that is small with respect to typical specimen dimensions. As in Small Scale Yielding in classical fracture theory [17], the crack is semi-infinite and lies on the negative x axis, as shown in Fig. 1. The boundary condition is that at distances far from the crack tip, the stress and deformation field approaches the usual inverse square root singularity of the classical elasticity solution. More details will be given in the next section.

The curvature induced Laplace pressure p_L resisting the opening of the crack (see Fig. 1) is related to the surface tension σ and the curvature of the deformed crack faces κ by:

$$p_L = \sigma\kappa \quad \kappa = \frac{v''}{\left[1+(v')^2\right]^{3/2}}, \tag{1}$$

where v is the crack opening displacement and a prime denotes differentiation with respect to x. In KSR, v' is assumed to be small everywhere and κ is approximated by:

$$\kappa = v''. \tag{2}$$

Due to this approximation, the governing equation describing the crack tip field is linear and the full machinery of analytic function theory can be used to formulate the crack problem. In contrast, TCL avoided this issue all together by placing a line force at the crack tip—the curvature induced Laplace pressure appears as a delta function in their formulation [13].

It is easy to understand why K_{local} had to be zero in KSR's analysis, that is, the local stress field near the crack tip cannot have an inverse square root singularity. Indeed, assuming K_{local} is positive, the crack opening for the upper crack face near the crack tip is:

$$v = \frac{2(1-\upsilon)K_{\text{local}}}{G\sqrt{2\pi}}\sqrt{-x}, \quad x < 0 \tag{3}$$

where υ is the Poisson's ratio and G the shear modulus. Substituting (3) into $\kappa = v''$ gives

$$\kappa \propto -K_{\text{local}}\,|x|^{-3/2}, \quad x \to 0^- \tag{4}$$

Eqs. (4) and (1) imply that the Laplace pressure has a *non-integrable* singularity; such a singular pressure field will induce an *infinite* negative stress intensity factor at the crack tip, which means that K_{local} goes to negative infinity. This is a contradiction to the original assumption that K_{local} is positive so the only possibility is $K_{\text{local}} = 0$, which implies that the stress is bounded or has a weaker singularity.

The fact that $K_{\text{local}} = 0$ brings up the possibility a line force can exist at the crack tip. Indeed, in linear elasticity fracture mechanics, the crack tip deforms into a cusp shape if the stress is bounded at the tip—the Dugdale-Barenblatt

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