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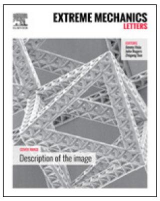

Static and dynamic elastic properties of fractal-cut materials

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ABSTRACT

We investigate the static and dynamic (phononic) elastic behavior of fractal-cut materials. These materials are novel in the sense that they deform by rotation of "rigid" units rather than by straining these units, can be fabricated by exploiting a simple cutting paradigm, and have properties that can be manipulated by control of the cut pattern and its hierarchy. We show that variation of fractal-cut level and cut pattern can be exploited to manipulate the symmetry of the elastic constant tensor, the elastic limit of deformation, and, therefore, the elastic response. By studying phonon behavior, we demonstrate how some cut symmetries naturally open acoustic band gaps. Several of the important features of the band structure can be directly related to the static elastic properties. Based upon our phonon calculations, we predict the acoustic transmission spectrum of an example fractal-cut structure and validated it through 3D printing and sound attenuation experiments.

1. Introduction

Heterogeneity in the structure of materials occurs commonly in natural materials and can be exploited in engineering materials to produce combinations of properties tailored for particular classes of applications. In most cases, the distributions of phases within materials exhibit a substantial degree of randomness. Exceptions include such engineered composites as laminates, aligned fiber composites, and woven structures. For many applications, it is desirable for the phases to have properties from opposite extremes; e.g., strong/brittle phases and weak/ductile phases. Other examples include cases where the elastic constants are either large or zero (i.e., holes). In this report, we consider the elastic (static and dynamic) properties of a uniform 2D material into which we introduce a periodic, hierarchical array of cuts; we refer to these as fractal-cut materials [1–3]. Such materials can be designed to produce a wide range of interesting and useful properties.

Consider the case of a square sheet with horizontal and vertical cuts. In the example shown in Fig. 1(a), a horizontal cut is made nearly across the sample leaving a ligament at the left and right edges and vertical cuts are made from the top and bottom surface to just short of the center, leaving ligaments at the center. When this structure is subject to nearly any external loading, it opens by rotation of section relative to other at the ligaments/hinges (Fig. 1(a)). In the limit that the ligament size goes to zero, the sections of the material between cuts remain undeformed during the rotation. Hence, we idealize this material as a set of rigid bodies, connected by hinges (in 2D or 3D) or possibly universal joints (in 3D). This process of separating the material into rotating blocks can be repeated in a hierarchical form by cutting

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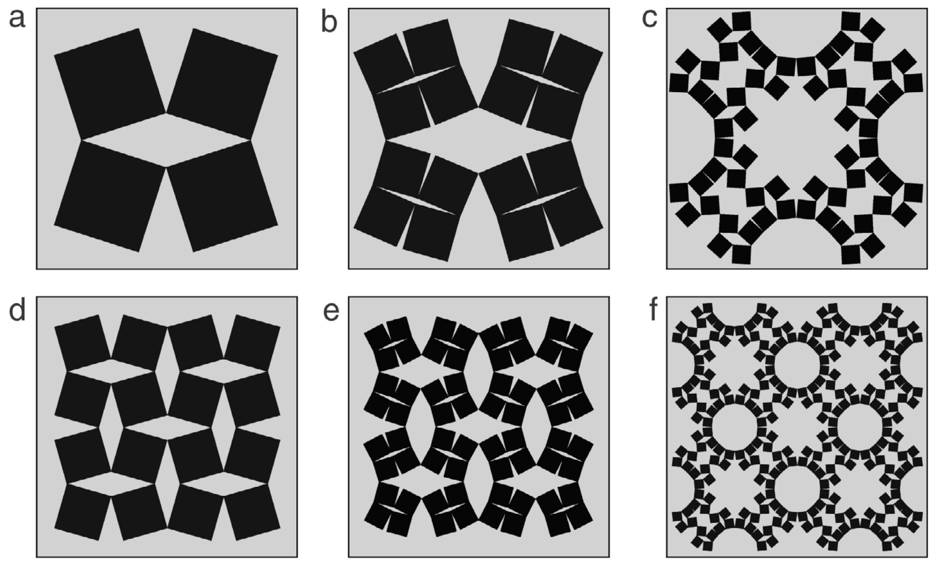

Fig. 1. Example of a (a) ''square unit cell'' that can be (d) repeated to create a ''square lattice''; a level 1 structure. Each square can be subsequently divided in exactly the same manner to create a (b) level 2 structure which may be (e) repeated to form a level 2 lattice. (c) and (f) show the corresponding level 3 structures/lattices.

the previously rigid/rotatable blocks. If this is done *ad infinitum*, the resultant structure is a fractal (see Fig. 1). Such structures have novel mechanical and dynamical properties associated with these two central features; rotatable units and structure across a range of scales.

Simple rotatable structures [4] can produce materials with negative Poisson ratios (i.e., auxetics) [5–7]; fractal-cut geometries can give rise to extremely large dilatations [1,2]. Such rotations are central to the auxetic behavior of several perovskite structures [8]. One natural consequence of the extremely large dilatations achievable with fractal-cut sheets is their ability to wrap non-zero Gauss curvature objects without wrinkling or tearing even when the matrix material is elastically rigid [1].

In addition to interesting elastic/geometric properties, fractal-cut materials also exhibit interesting dynamic (phononic) properties. Since our fractal-cut materials are periodic, there is a possibility that such structures may exhibit band gaps in the phonon spectrum (i.e., frequency ranges in which sound does not propagate); such periodic metamaterials are known as phononic crystals. The phononic band structure is dictated by the acoustic properties of the constituent materials and their spatial distribution (in the present case, one phase is solid and the other is air/vacuum). Unit cell geometries that have been exploited in the design of phononic crystals vary from very simple [9] to complex [10], including fractals [11–13].

The most obvious application of phononic crystals is in sound insulation (e.g., [14]); however, the high cost of manufacturing such periodic structures makes most such applications untenable. However, acoustic metamaterials have a number of other interesting applications including waveguides [15] in which sound can be channeled into different directions with little energy loss, phononic lenses [16] akin to optical lenses, and devices which exploit negative refraction [17].

One important challenge in acoustic applications is to design metamaterials such that the band gaps fall into the frequency range of interest. For acoustic applications, this is below 1 kHz. However, since vibrational frequencies scale inversely with unit cell size; 1 kHz commonly implies meter length scales. Such scales are usually impractical; hence, an important challenge is to design materials with wide, low frequency band gaps and of reasonable size. While recent advances based upon resonant structures have pushed band gaps into the desired low frequency region [18], the resultant band gaps are too narrow for many applications. Since fractal-cut materials can be designed with tunable elastic properties, we investigate here how such materials can be designed to manipulate acoustic properties. As we demonstrate below, elastic properties can be used as a simple predictor of several important features of the dynamical response of these materials.

In this paper, we report how the elastic (static and dynamic) properties of such rotatable, fractal-cut structures vary with degree of hierarchy, cut-pattern symmetry, and hinge properties. We begin with an analysis of the geometric properties of rotatable hinge, fractal-cut structures and then analyze the impact of finite hinge stiffness in order to predict the elastic behavior (elastic constants) of such structures. Since real fractal-cut materials will always have finite ligament sizes, we perform finite element analyses in order to determine the correlation between hinge stiffness and ligament geometries in elastic materials (where both the ligament and blocks are made from the same material). Next, we consider dynamic elastic properties by analyzing sound propagation through these materials—determining the phonon band structure. We then exploit 3D printing to manufacture a fractal-like structure and measure its acoustic properties to compare with our predictions. Finally, we examine the correlation between the static and dynamic elastic results in order to provide guidance for the design of such materials.