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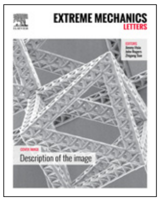

Exponential stress mitigation in structured granular composites

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ABSTRACT

Granular media has been used throughout history as rudimentary yet effective impact mitigation. The unique response of natural granular media is associated with the existence of a network of stress propagation pathways, i.e. a force chain network, which spatially and temporally redirects and moderates the impulse. A variety of structured materials have been proposed to improve the impact mitigating properties compared to natural systems. However, these engineered materials use permanent deformation or viscoelastic properties to dissipate energy, generally limiting their lifetime or effective frequency and temperature range. Here, we take inspiration from natural granular media to engineer a structured composite that exhibits an exponentially fast decay of the leading transmitted pulses. The ordered network geometry allows for an analytical description of the transmitted pulses, which we validate though experiments and numerical simulations. In contrast to other structured materials used for impact mitigation, these networks exhibit reversible deformation, function over all frequencies, and possess a low relative density. Our results open new possibilities for the design and realization of increasingly complex material systems with engineered stress wave transmission pathways.

1. Introduction

Nowadays, material microstructures can be precisely controlled at different scales to achieve new and improved mechanical properties [1–3]. Here we take inspiration from the advantageous properties of natural granular media, to design and build a structured composite material that employs granular chains to control stress propagation pathways. Granular chains present a unique dynamic response due to both their discrete nature and the nonlinear, Hertzian [4], interaction between particles under compression. For the special case of a uniform particle chain, any axial excitation results in the formation and propagation of nonlinear compact pulses, or solitary waves [5–7]. More generally, granular chains dictate the dynamic behavior of natural, disordered granular packings [8,9]: instead of the uniform, linear wave propagation observed in a homogeneous solid, a granular medium composed of the same material will transmit excitations through a complex network of force chains [10–15], *i.e.* preferred loading paths based on the inter-particle contact network (Fig. 1(a)). This underlying granular chain network controls the response to static forces [10,11], resistance to intruder impacts [15,16], and acoustic wave transmission [13,14]. In particular, the amplitude of dynamic excitations are observed to decay exponentially with distance from the impact [13,15,17], which makes granular materials highly attractive for wave mitigation applications. However, the primary mechanism for this decay and the local distribution of pulse amplitudes within a granular system are still being investigated.

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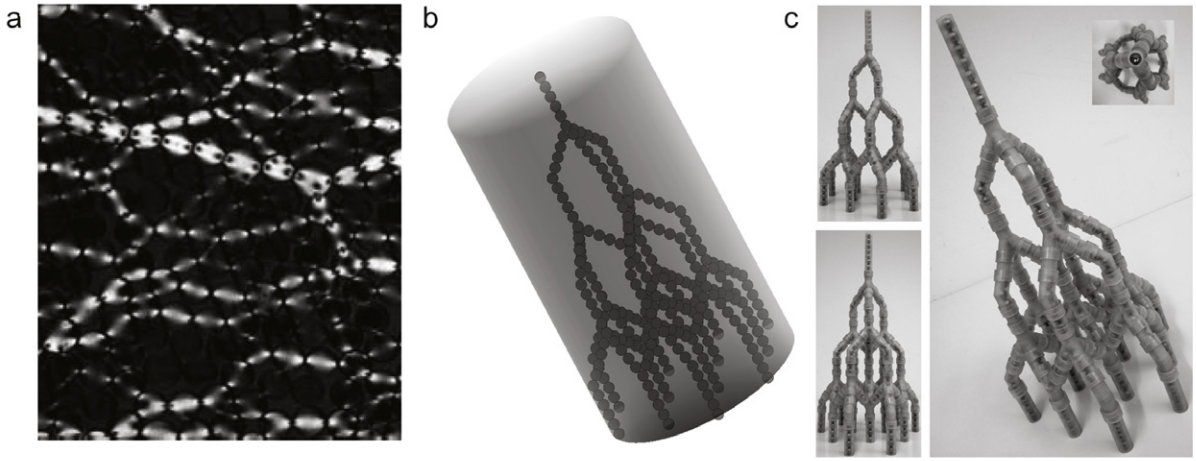

Fig. 1. Granular networks. (a) Example of force chain network in disordered granular packings (image reproduced and adapted with permission from [11]). The light regions within the packing of photoelastic cylinders indicate higher stresses. (b) Conceptual schematic showing the engineered granular network, gray spheres, fully embedded in a supporting matrix, shaded cylinder. (c) Photographs of the actual granular network used in experiments. To minimize 3D printed material usage, the supporting matrix was reduced to only the necessary confining channel structure.

Using an engineered granular network to design structured materials both permits control over the wave propagation pathway, and simplifies the dynamic response allowing for predictive analytical descriptions of the leading pulses. We predict and experimentally observe an exponential decay of the leading pulse amplitudes through our engineered granular network (Fig. 1(b) and (c)). This engineered network provides insight into the dynamics of natural granular media and allows for the design of new, more efficient stress wave mitigating structures.

The exponential decay rate of the maximum transmitted pulse amplitude is a result of the branched material structure in combination with the nonlinear dynamic response. The branched structure allows for spatial mitigation, and the nonlinear dynamics for temporal mitigation. Under quasi-static loading, the transmitted amplitudes in each segment can be easily calculated from the symmetric amplitude splitting; while the edge (smallest) amplitudes decay exponentially, the central (largest) amplitude decays faster than linear but slower than exponential. Additionally, the transmitted amplitudes for a network geometry composed of a linear media would not differ between quasi-static and dynamic loadings (in the simplest scenario of a non-dispersive media and neglecting edge effects). The amplitude dependent wave speed in our system [5–7], a property deriving form the nonlinearity, reduces the occurrence of interior wave recombinations, resulting in added temporal wave mitigation of the largest transmitted pulse compared to linear systems. This exponential decay could also be observed in other nonlinear, *i.e.* non-granular, materials. However, the advantage of using granular chains lies in their inherent ability to break up large amplitude or long duration excitations into a series of smaller pulses [7]. Here, we focus on the relationship between the structured branching geometry and the global dynamic properties in homogeneous granular networks.

The network geometry studied here (Fig. 1(b) and (c)) was chosen to capture the fundamental physical mechanisms relevant for a general network structure, *i.e.* multiple wave splittings, bends, and combinations. However, our approach is general and could be used to describe and design an arbitrary force chain structure with variable particles sizes, materials, and network geometries. Previous studies have observed the pulse splitting and combining mechanisms individually using 2D Y-shaped granular systems [18–21]. Here, we investigate a 3D branching geometry, which gives rise to multiple occurrences of each mechanism, and investigate the overall stress transmission properties emerging in these types of systems. This study both provides a better comparison with the complex force chain structures observed in natural granular media, and allows for improved mitigation performance compared to the 2D systems recently investigated [22]. Our engineered network incorporates uniform chains of particles only along the predetermined force chain pathways with a supporting matrix filling the remaining 3D volume (Fig. 1(b)). This results in a structured material presenting a low effective density compared to densely packed granular media.

2. Materials and methods

2.1. Experiments

In experiments, the granular networks were constructed from stainless steel sphere chains (type 440C from mcmastercarr.com) held in place by a polymer supporting channel structure, as shown in Fig. 1(c). The stainless steel particles were assumed to have an Young's modulus of $E = 200$ GPa, Poisson's ratio of $\nu = 0.28$, and a density of $\rho = 7800$ ${}^{\mathrm{kg}}/{}_{\mathrm{m}^3}$ [22]. The particles used in experiments have a manufacturer specified radius of $R = 4.7625$ mm. The printed VeroClear material used for the supporting channels has a manufacturer specified $\rho = 1045$ ${}^{\mathrm{kg}}/{}_{\mathrm{m}^3}$ and Young's modulus $E = 2$–3 GPa (http://objet.com).

For ease in assembly, the supporting channel structure was 3D printed in a modular fashion and then assembled into networks of variable size. To generate a single incident pulse, each granular system was excited by dropping