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Advanced functionally graded plate-type structures impacted by blast loading

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ABSTRACT

The foundation of the theory of functionally graded plates with simply supported edges, under a Friedlander explosive air-blast are developed within the classical plate theory (CPT). Within the development of the theory, the two constituent phases, ceramic and metal, vary across the wall thickness according to a prescribed power law. The theory includes the geometrical non-linearities, the dynamic effects, compressive tensile edge loadings, the damping effects, and thermal effects. The static and dynamic solutions are developed leveraging the use of a stress potential with the Extended Galerkin method and the Runge–Kutta method. Validations with simpler cases within the specialized literature are shown. The analysis focuses on how to alleviate the unwanted effects of large deformations through proper material selection and the proper gradation of the constituent phases or materials.

1. Introduction

During combat situations, the structure of army military vehicles may have to structurally endure the effects of blast loading. Advances in functionally graded materials (FGM) which combine the properties of two dissimilar materials has been a motivating factor in viewing these types of materials as a viable alternative to the current isotropic metallic structures being utilized within the army's military vehicles. Practical examples of where these FG plate-type structures could be used include: (1) the underbelly of the vehicle referred to as the hull, or (2) side plate armor on the vehicle to prevent the effects of blast and penetration. FGM's are microscopically nonhomogeneous with thermo-mechanical properties which vary smoothly and continuously from one surface to another. These graded structures allow the integration of dissimilar materials like ceramic and metals that combine different or even incompatible properties such as hardness and toughness.

In this paper, the foundation of the nonlinear theory of functionally plate-type structures under an explosive air-blast is developed. An approximate solution methodology for the intricate nonlinear boundary value problem is devised, and results that are likely to contribute to a better understanding of the structural behavior under an explosive blast with beneficial implications toward their improved design and exploitation are presented.

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2. Basic assumptions and preliminaries

The plate mid-surface is referred to a cartesian orthogonal system of coordinates (x, y, z), where z is the thickness coordinate measured positive in the upwards direction from the mid-surface of the plate with h being the uniform plate thickness of the plate, and y is directed perpendicular to the x-axis in the plane of the plate. See Fig. 1 below.

The nonlinear elastic theory of FG Plates is developed using the classical plate deformation theory [6]. It is also assumed that the FG plate is made-up of ceramic and metal phases whose material properties vary smoothly and continuously across the wall thickness. By applying the rule of mixtures, the material properties such as Young's Modulus, Density, and Poisson's Ratio are assumed to vary across the wall thickness as

$$P(z) = P_c V_c(z) + P_m V_m(z), \tag{1}$$

in which P_c and P_m denote the temperature-dependent material properties of the ceramic and metallic phases, of the plate, respectively, and may be expressed as a function of temperature [7,8] as

$$P = P_0\left(P_{-1}T^{-1} + 1 + P_1 T + P_2 T^2 + P_3 T^3\right). \tag{2}$$

P_0, P_{-1}, P_1, P_2, and P_3 are the coefficients of temperature $T(K)$ and are unique to the constituent materials. $V_c(z)$ and $V_m(z)$ are correspondingly, the volume fractions of the ceramic and metal, respectively, fulfilling the relation

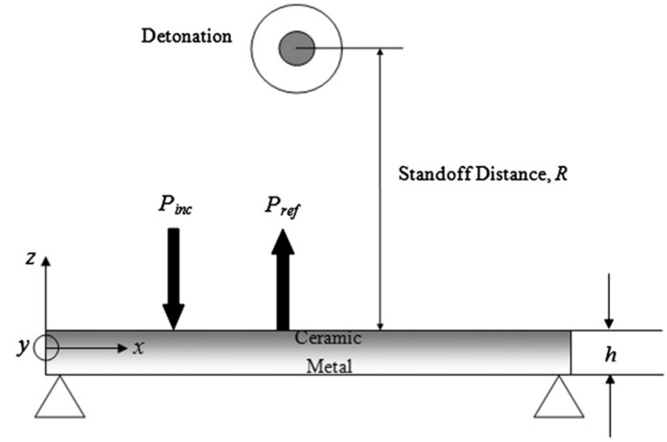

Fig. 1. A simply supported functionally graded plate shown in 2-D under an explosive blast.

$$V_c(z) + V_m(z) = 1. \tag{3}$$

By virtue of Eq. (3), Eq. (1) can be expressed as

$$P(z,T) = [P_c(T) - P_m(T)]V_c(z) + P_m(T). \tag{4}$$

By observation, one can deduce that for $V_c(z) = 0$, $P(z,T) = P_m(T)$ and for $V_c = 1$, $P(z,T) = P_c(T)$. As a result, $V_c(z) \in [0, 1]$.

Two scenarios of the grading of the two basic component phases, ceramic and metal, through the wall thickness are considered.

Case (1). (*The phases vary symmetrically through the wall thickness, in the sense of having full ceramic at the outer surfaces of the plate and tending toward full metal at the mid-surface. For this case,* $V_c(z)$ *can be expressed as*)

$$V_c(z) = \left(\frac{z}{h/2}\right)^N \left(\frac{1+\mathrm{sgn}(z)}{2}\right) + \left(\frac{-z}{h/2}\right)^N \left(\frac{1-\mathrm{sgn}(z)}{2}\right), \tag{5}$$

where the Signum function is defined as

$$\mathrm{sgn}(z) = \begin{cases} 1, & z > 0 \\ 0, & z = 0 \\ -1, & z < 0 \end{cases}. \tag{6}$$

N is termed the volume fraction index which provides the material variation profile through the plate wall thickness, $(0 \le N \le \infty)$. A pictorial representation of the distribution of the constituent materials is shown below in Fig. 2.

Case (2). (*The phases vary non-symmetrically through the wall thickness, and in this case there is full ceramic at the outer surface of the plate wall and full metal at its inner surface. For this case,* $V_c(z)$ *can be expressed as*)

$$V_c(z) = \left(\frac{h+2z}{2h}\right)^N. \tag{7}$$

Below is a pictorial representation of the antisymmetric case shown in Fig. 3.

It should be noted that in contrast to case (2), where there exists coupling between stretching and bending, such coupling is not present for the symmetric case (1). Also, for the purposes of simplicity the Poisson's ratio will be assumed to be constant throughout the plate structure. From Eqs. (1)–(7), the effective material properties of a FG plate can be expressed for the asymmetric case as

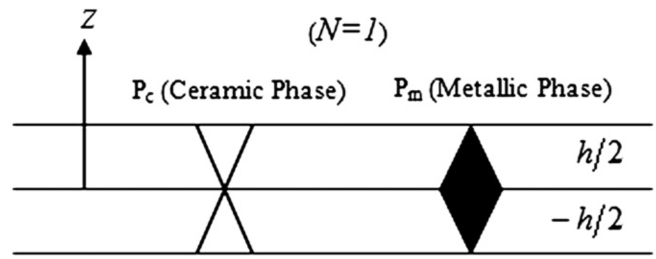

Fig. 2. Distribution of the constituent materials through the plate thickness for the symmetric case.

$$[E(z,T), \rho(z,T), \alpha(z,T)] = [E_{cm}(T), \rho_{cm}(T), \alpha_{cm}(T)]\left(\frac{h+2z}{2h}\right)^N + [E_m(T), \rho_m(T), \alpha_m(T)] \tag{8}$$

and for the symmetric case as

$$[E(z,T), \rho(z,T), \alpha(z,T)] = [E_{cm}(T), \rho_{cm}(T), \alpha_{cm}(T)]\left[\left(\frac{z}{h/2}\right)^N \left(\frac{1+\mathrm{sgn}(z)}{2}\right) + \left(\frac{-z}{h/2}\right)^N \left(\frac{1-\mathrm{sgn}(z)}{2}\right)\right] + [E_m(T), \rho_m(T), \alpha_m(T)] \tag{9}$$

$$\nu(z,T) = \nu(T), \tag{10}$$

where

$$E_{cm} = E_c - E_m, \rho_{cm} = \rho_c - \rho_m. \tag{11}$$

3. Kinematic equations

3.1. The 3-D displacement field

Consistent with the classical plate theory [6], the distribution of the 3-D displacement quantities through the wall thickness can be expressed as

$$u = u_0 - z w_{0,x} \tag{12a}$$

$$v = v_0 - z w_{0,y} \tag{12b}$$

$$w = w_0. \tag{12c}$$

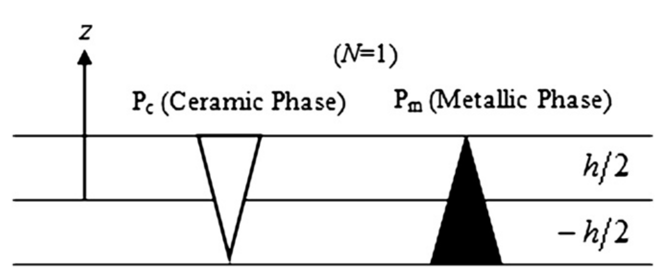

Fig. 3. Distribution of the constituent materials through the plate thickness for the antisymmetric case.