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Integral design of contour error model and control for biaxial system

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ABSTRACT

This paper focuses on the contour following accuracy improvement for biaxial systems using cross-coupled control (CCC). It proposes an integral design method including contour error model, contour control effort distribution and the CCC algorithm. First, a contour error model using the contour algebraic equation and its partial derivatives is established without the small tracking error assumption. This model satisfies the condition that it equals to zero if and only if the real contour error value vanishes, which makes perfect contour following become possible in theory. Then, in order to decouple the contour following the feed-direction tracking, contour control effort distribution is decided to be in line with the normal vector at the desired point. Through expanding the proposed contour error model with Taylor series to make it be related to tracking errors of both axes, the stability condition of CCC is analyzed by the contour error transfer function (CETF). Experiments are carried out on an X–Y motion stage to verify the proposed method. The results show that it improves the contour following accuracy greatly in various conditions, even when large tracking errors occur.

1. Introduction

In computer numerical control machines, contour accuracy is one of the most critical indices which determine the product surface quality. Contour error, which is defined as the nearest distance from the actual point to the desired trajectories, is used to characterize contour accuracy. It is noted that in perfect tracking situations each axis follows its desired trajectory exactly and the contour error is zero. Thus, a natural idea to improve the contour accuracy is reducing individual axis tracking errors. However, perfect tracking rarely occurs due to external disturbances and the tracking errors are usually of non-zero. Chiu [1] found that contour errors did not necessarily decrease with tracking errors reducing and gave a set of data that had smaller tracking errors but larger contour ones. Actually, Poo [2,3] analyzed that some factors including individual axes tracking errors, dc gain mismatch in feedback control, etc. together affected the contour accuracy. In reality, such negative factors cannot inevitably be handled ideally. Therefore, in order to improve contour accuracy, additional efforts should be devoted to contour following control.

Till now, contour control has been an active research field and a large number of publications can be found [4–9]. Existing contour control strategies mainly fell into two categories, task coordinate frame approach (TCFA) and cross-coupled control (CCC). In TCFA scheme, a task coordinate frame is established on the desired contours. Then, it transforms the system dynamics into feed-direction tracking and contour-direction tracking. At last, decoupled control algorithm is designed in each direction and the individual axis control effort is gained by inverse transformation. Chiu [1] constructed the task coordinate frame on the desired point and contour-direction tracking errors were the projection of the tracking errors on normal and bi-normal directions. Hu [10] utilized adaptive robust control algorithm to control such transformed dynamics, compensating for the disturbances and achieving good results. In [11], different from [1,10], the contour-direction tracking error was denoted by a contour index, i.e., a distance from the actual point to the tangential circle on the desired point in the normal direction. And this contour index is handled by a sliding mode control algorithm. Chen [12] used $n-1$ algebraic equations as equivalent errors to describe the contour-direction tracking errors for n-axial systems. Together with the tangential error, these equivalent ones were dealt with integral sliding mode control algorithm. An advantage was that the equivalent errors became zero if and only if real contour ones vanished. Sencer [13,14] built the task coordinate on a point whose delayed time to the desired point was estimated by the feed-direction tracking error dividing feed rate. The contour-direction errors were the normal and bi-normal components of tracking errors on such point. Yao [15] proposed a global orthogonal task coordinate frame for biaxial system and the contour-direction

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errors were described by the contour algebraic equation dividing its partial derivatives at the actual point, which was a first-order approximation of the real contour error and had the similar advantage as those in [12]. Based on this construction, adaptive robust control was utilized, improving the contour following accuracy greatly especially in those points with large curvature. In [16], Lou created a task polar coordinate frame with radius and angular direction. The contour-direction errors were estimated by the tangential circle on the desired point, which was a second-order approximation. In all the above TCFAs, transformation and inverse transformation were contour-dependent, which might bring time-variety to systems during movements.

In CCC scheme, each axis is controlled by both tracking controller and contour controller. The tracking one is designed independently to gain satisfactory tracking performance, while the contour one is to achieve better contour performance. That is, the tracking controllers can be designed individually to "pull" the system from the actual point to the desired one; while the contour one is to couple all the axes and force the system from the actual point to the contour, instead of the desired point [17]. Due to its simplicity, CCC received more and more attention and became popular in contour control. Key issues in CCC structure are the contour error calculation, the contour control effort distribution to each axis and the contour control algorithm. From the definition, contour error is the shortest distance from the actual point to the desired path. However, it is almost impossible to get real contour error during a control sampling period due to its computation complexity [18], except for some specific desired paths, for example, linear line and circle. Therefore, in applications, it usually uses an approximated approach to build a contour error model for CCC. In [19], the shortest distance from the actual point to the tangential line on the desired point was proposed as a contour error model. It was a linear combination of individual axis tracking error with varying coefficients depended on the desired contours. These coefficients determined the contour control effort distribution to each axis. Actually, the contour-direction error in [1,10] belonged to this type of model. Yeh [20] used this tangential line based contour error model and gave a contour error transfer function for stability analysis. This model was also used together with adaptive control [21], observer [22], iterative learning control [23], and predictive control [24] to achieve better contour following performance. Cheng [25] proposed a contour error model by the distance from the actual point to the line passing through desired point and delayed desired point as in [13], which was suitable for free-form path. The contour control effort distribution was determined by this line slope. In [26], together with the enhanced position error compensator, this model was controlled by a proportional CCC to achieve improved contour accuracy. Huo [27] proposed a contour error model through line-segment approximation of the original contour with a number of reference positions generated by the CNC interpolator and the contour error was compensated by a generalized Taylor series expansion approach. Yang [28] gave the contour error model as the shortest distance from the actual point to the tangential circle on the desired point, denoted as tangential circle based contour error model. The contour control effort contribution was determined by the line passing through the tangential circle center and the actual point. This model can be written as a linear combination of tracking errors plus a curvature-related item. The CCC was in position loop and the stability was analyzed via small gain theory. Since it belonged to second-order approximation, it was expected to perform better than the tangential line based contour error model and experiment results verified it. Huo [29] estimated the contour error based on piecewise liner approximation, which is the shortest distance from actual point to $N-1$ lines passing through the consecutive two reference points. Zhu [30] proposed a second-order approximation as the contour error model. Different to that in [28], this model was purely a linear combination of tracking errors. Simulation and experiment results showed that it had the least approximation error compared to other models. It is noted that contour-direction tracking errors in TCFA can be used as the contour error models, for example, contour index in [11] and the contour equation dividing its partial derivatives in [15]. But CCC algorithms using the models in [11,15,30] have not been found yet.

It is necessary to point out that feedback controller is designed to vanish its input, i.e., individual tracking controller is to make tracking error be zero while CCC is to make contour error model instead of contour error be zero. Thus, in order to achieve perfect contour accuracy in theory, it must suffice that contour error model is zero if and only if the real contour error is zero. The above contour error models except that in [15] did not satisfy this condition. Although CCC on these models improves contour following accuracy in some sense, it cannot gain zero contour error in theory. The model in [15] had the advantage that it is zero if and only if contour error is zero. But it is only used in TCFA and researches on CCC remain open. This paper proposes a contour error model satisfying the above condition and determines the contour control effort contribution decoupling with feed-direction tracking control for biaxial systems. By expanding this model via Taylor series, the stability condition of CCC structure is analyzed. Experiment results show that it achieves contour accuracy improvement even when the tracking errors are large.

The remainder of this paper is as follows. Section 2 proposes the contour error model. Section 3 gives the CCC for this model and analyzes the stability. Experiments are carried out in Section 4 and at last Section 5 concludes this paper.

2. Contour error model

2.1. Problem formulation

In fulfilling the manufacturing tasks, biaxial systems are usually controlled to move along a desired contour which is constrained by a known equation

$$f(x, y) = 0 \quad (1)$$

In traditional methods, each axis is controlled individually to track its trajectory. Due to various external disturbances and mismatch between two axes, there inevitably exist tracking errors and contour errors. For example, Fig. 1 shows a schematic diagram about the tracking error and contour error at a moment in the movements. The desired point is p_d, while p_{a1} and p_{a2} are possible actual points. The tracking error of p_{a1} is $p_d - p_{a1}$, which can be denoted by e_1. The point p_{c1} is the nearest points on the desired contour to point p_{a1}. Thus, the contour error of point p_{a1} is $p_{c1} - p_{a1}$, which can be denoted by ε_1. Similar denotations are about point p_{a2}. That is, the tracking error and contour error of point p_{a2} are e_2 and ε_2 respectively. From Fig. 1, it is clear that p_{a2} has larger tracking error but smaller contour error. Hence, reducing tracking error by individual axis tracking controller does not necessarily improve contour accuracy. Thus, in order to achieve precise contour following, it should turn to contour control approaches.

CCC is a popular contour control method whose purpose is to reduce contour error while not to affect the feed-direction tracking performance. Under CCC, the actual point will be 'pulled' to the desired contour. When using CCC, three issues should be concerned.