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On the use of variational inequalities to model impact problems of elasto–plastic media

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Abstract

A new variational inequality-based formulation is presented for treating dynamic elasto–plastic contact problems. The incremental variational inequality representing this class of problems is developed in an updated Lagrangian framework. A new technique for representing the kinematic contact conditions, based on C^1-continuous spline interpolation and an intermediate surface with uniquely defined normal, is developed. The solution algorithm is based upon the iterative use of mathematical programming and Lagrange multipliers to identify the candidate contact surface and contact stresses. This approach guarantees the accurate imposition of the active kinematic contact constraints and avoids the use of special contact elements. The dynamic variational inequalities formulations for the two sub-problems are solved using the generalized-α time integration scheme. The solution strategy accounts for the effect of friction through the use of an appropriate regularization technique in the virtual work expressions. This newly developed approach leads to a significant reduction in numerical oscillations in impact and dynamic frictional problems, and is less sensitive to variations in the time increment. It also reduces the number of iterations needed to achieve convergence. The robustness and accuracy of the proposed FE algorithm are demonstrated by application to a number of case studies.

Keywords: Elasto–plastic; Contact; Dynamic; Impact; Variational inequality; Large deformations; Generalized-α scheme; Friction

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1. Introduction

Current finite element codes are based on the use of the traditional variational formulations to treat dynamic contact problems. These formulations usually rely on the use of contact elements that involve user-defined parameters, which affect the convergence and accuracy of the solution. However, frictional contact problems can be most accurately formulated in terms of variational inequalities. This is because variational inequalities provide an accurate representation of physical problems involving unilateral constraints [1,2]. The advantages of using variational inequalities in the formulation of contact problems lies in the robustness of their treatment, especially when frictional effects are included [3].

In earlier studies, Meguid and his collaborators treated elastic [4,5], and elasto-dynamic [6] contact problems using appropriate variational inequalities formulations, contact search and solution algorithms. In this work, we devote our attention to the treatment of dynamic elasto–plastic frictional contact problems. Specially, attention is devoted to the development of the kinematic contact conditions which are based on cubic-spline interpolation and an intermediate surface with uniquely defined normal vectors. Currently, the contact surfaces are generally defined as a sequence of lines (or curves) connecting the FE nodes with mostly C^0-continuity. Typically, the contact surfaces are non-smooth at the exterior nodes even for higher order elements. In this case, contact surfaces violate the assumption of smoothness, which are essential to the proofs of uniqueness and convexity of contact problems. In order to limit these inconsistencies, contact surfaces are commonly modeled by employing a disproportionately fine mesh at the vicinity of the contact region.

Based on these surface interpolations, several classes of contact search algorithms can be distinguished for the evaluation of kinematic contact conditions [7]. The node-to-node algorithm requires identical meshes and coincident nodes (Fig. 1a). A contact constraint can be applied between each pair of coincident nodes. In the node-to-segment approach, the nodes of one surface (commonly referred to as the master) are prohibited from penetrating the second surface (commonly referred to as the slave) (Fig. 1b). Therefore, the no-penetration contact constraints are enforced only at discrete points on the master surface [8]. A modification of the node-to-segment approach involves enforcing the contact constraint between the integration points of the master and the slave surfaces [9]. The two-pass node-to-segment contact algorithms eliminate the bias between master and slave surfaces by performing the nodal contact search twice (Fig. 1c) [10]. Another approach utilizes an intermediate contact surface technique developed between the two contacting bodies (Fig. 1d). The nodes from each contact surface are projected orthogonally to the other contact segment [11]. However, discretization of the contact surfaces does not provide smooth inter-element connections, even for higher order elements.

To overcome this limitation, the use of cubic splines was considered (see, e.g., [12]) to represent contacting surfaces. In this case, the normal vectors are defined based on the projection of nodes from one surface to the other. A new technique that employs spline interpolation to enforce the kinematic contact conditions, and guarantees C^1-continuity, with uniquely defined normal vectors, is proposed using an intermediate cubic spline surface. This approach has a specific merit in treating dynamic problems where contact surfaces vary with time. It ensures that less number of iterative updates of normal vectors is used to reach convergence.