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A pole/zero cancellation approach to reducing forced vibration in end milling

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ABSTRACT

An analytical model for the forced vibration in an end milling process is derived and a criterion in selecting cutting parameters to reduce the forced vibration is presented in this paper. The analytic expression for the forced vibration due to the periodic milling force is obtained as the product of the Fourier transform of the milling force and the frequency response function of the structure dynamics. The pole/zero cancellation technique is then employed for reducing the forced vibration. Analysis shows that the suppression of forced vibration can be achieved by choosing cutting parameters so that one of the zeroes of the Fourier transform of the milling process function is near the pole of the structure dynamics. A design equation in terms of cutter geometry, axial depth of cut, spindle speed and structure resonant frequency is derived for the conditions when the forced vibration can be minimized. The presented analysis is illustrated through numerical simulation and verified by experimental results.

1. Introduction

Vibration in a milling process causes poor surface finish and reduces the life expectancy of the cutter and various components of the machine. Causes of vibration need to be analyzed thoroughly so that guidelines for the proper operation of the milling process can be prescribed. The occurrence of vibration in milling can be attributed to two major sources: the self-induced vibration or chatter and the forced vibration. Chatter is the unstable transient vibration in the cutting process and mainly caused by the regenerative and mode-coupling effects [1,2]. The explicit analytical expressions for the chatter limits have been proposed [3–6] to aid in the selection of appropriate depth of cut and spindle speed, etc. to avoid the occurrence of chatter. On the other hand, forced vibration is the steady state vibration due to the periodic excitation of the intermittent cutting engagement of the milling cutter on the flexible cutter/workpiece structure, and it almost always exists even in the absence of chatter. While a great deal of research has been done on the chatter analysis, less attention has been paid to the forced vibration. One reason for this could be due to the complexity encountered in analyzing the milling force pulsation as many cutting parameters are involved in the process. Another reason might be due to the fact that most forced vibration is not as severe compared to the unstable chatter vibration. Although the forced vibration is generally relatively smaller than the chatter vibration, it still could significantly affect the machining qualities especially when milling high flexible structures, such as the blade. The strong forced vibration will be generated when the force harmonics are near the resonant frequency of flexible structure. If this vibration is not well controlled, it will result in overcut of the surface, and thus lead to poor dimensional accuracy. Therefore, the avoidance of the forced vibration plays an important role in improving the machining qualities of the milling process.

So far, the numerical approaches have been successfully adopted in many studies [7–13] to model the structure vibration in the investigation of system dynamics and surface errors of milling processes. Although numerical method offers a convenient, direct means of finding the steady state response to the forced excitation of the periodic milling forces as well as the system transient response as related to the stability analysis, searching large parameter domains in this manner is inefficient. Therefore, analytical models to milling problems provide an attractive alternative. Wang et al. [14] presented an analytical model for the force pulsation in the milling process. In this model, the Fourier coefficients of the periodic forces are expressed as the product of the Fourier transforms of three process functions, and each process function is dominated by the different cutting parameters. This frequency domain model is helpful to distinguish the contribution of each cutting parameter on the milling forces. Mann et al. [15] indicated that the regenerative effect is absent in stable machining. Schmitz and Mann [16] neglected the regenerative effect and then proposed a closed-form solution for the forced vibration by combination of the frequency domain models for the periodic milling forces and for the structure dynamics.

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Nomenclature

α	cutter helix angle
β	radial angular position of cutting point on the cutter
β_a, β_p	radial angle of axial immersion and the flute spacing angle
cwd, CWD	chip width density function and its Fourier transform
d_a, d_r	axial and radial depths of cut
d_x, d_y, D_x, D_y	displacement functions in the X and Y directions and their Fourier transforms
f_x, f_y, F_x, F_y	cutting forces in the X and Y directions and their Fourier transforms
ϕ	cutter angular displacement
G_i	transfer function of the vibration system
H_x, H_y	frequency response functions of the structure dynamics in the X and Y directions
η_p	frequency ratio
k_t, k_r	tangential and radial cutting coefficients
m_i, k_i, c_i	modal mass, stiffness and damping
N	flute number of the cutter
p_x, p_y, P_x, P_y	elementary cutting functions in the X and Y directions and their Fourier transforms
p_{xx}, p_{xy}, p_{yx}, p_{yy}	elementary cutting functions accommodating the feeds in the X and Y directions
θ, θ_1, θ_2	angular position of cutting point in the workpiece, entry and exit angles
R	cutter radius
T	period of spindle revolution
T_n	resonant period of structure
ts, TS	tooth sequence function and its Fourier transform
t_x	feed per tooth
w_r	radial cutting window function
Ω	frequency of spindle revolution
ω, ω_n, ω_0	frequency, structure resonant frequency and normalized frequency
ω_{0z}	frequency of the first zero of CWD function

By replacing the force model in [16] with that in [14], this paper uses a different analytical expression for the forced vibration. Based on this vibration model, the method for choosing the cutting parameters to reduce the forced vibration is then discussed. Adjusting the spindle speed has been the most intuitive, common practice in avoiding such resonance. However, by exploring the analytical force model, this paper shows that the Fourier transform of the process function has characteristic zeros at some specific frequencies, and reduction of forced vibration can be achieved by a pole/zero cancellation technique, i.e. by designing these zeros to be near the resonant frequency of the structure dynamics. The presented vibration model and the proposed method for reducing vibration is illustrated by numerical simulations and verified through milling experiments.

2. Modeling of milling process

A cylindrical coordinate system, $\beta-r-h$, is attached to the cutter to represent the geometry of the active cutting edges, as illustrated in Fig. 1 for an end mill. The h axis of the coordinate coincides with the cutter rotation axis, and the origin is located at one end of axial depth of cut with positive h pointing to the other end. The radial angular position of cutting point, β, at $h=0$ on an arbitrarily chosen first cutting edge is defined to be $\beta=0$, and is related to h by $h=R\beta/\tan\alpha$, where R is the cutter radius and α the helix angle of the cutter. A work coordinate system, X–Y–Z, is attached to the workpiece. It is assumed that the cutter moves in the X direction at a feed rate of t_x per tooth. In addition, an angular position variable θ in the X–Y–Z system is defined to represent the angular position of the cutting point in the workpiece. The angular displacement of the cutter with respect to the X–Y–Z frame is represented by $\phi=2\pi\Omega t$, where Ω is the spindle frequency.

It was shown in [14] that the milling forces for an end mill, without considering structure vibration can be modeled as the convolution of three process functions, including the tooth sequence function, ts, the chip width density function, cwd and the elementary force functions, p_x and p_y. That is

$$\begin{Bmatrix} f_x(\phi) \\ f_y(\phi) \end{Bmatrix} = ts(\phi)*cwd(\beta)*\begin{Bmatrix} p_x(\theta) \\ p_y(\theta) \end{Bmatrix} \tag{1}$$

where $*$ stands for convolution operation. The tooth sequence function represents the periodic engaging sequence of the rotating cutter and is written as

$$ts(\phi) = t_x \sum_{k=-\infty}^{\infty} \delta(\phi-(k-1)\beta_p), \quad \beta_p = \frac{2\pi}{N} \tag{2}$$

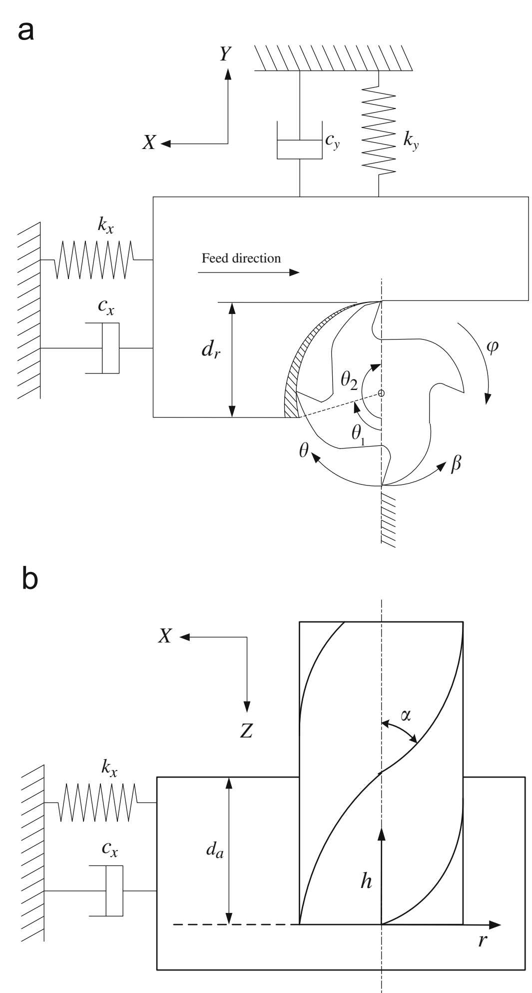

Fig. 1. Coordinate systems and cutting geometry for an end milling process.