International Journal of Machine Tools & Manufacture 48 (2008) 1335–1344

Contents lists available at ScienceDirect

International Journal of Machine Tools & Manufacture

journal homepage: www.elsevier.com/locate/ijmactool

A stochastic grinding force model considering random grit distribution

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ARTICLE INFO

Article history:
Received 15 February 2008
Received in revised form
20 May 2008
Accepted 25 May 2008
Available online 11 June 2008

Keywords:
Random grit distribution
Stochastic grinding force model
Grit density function
Power spectrum density
Convolution force model

ABSTRACT

Incorporating the random nature of grit distribution, this paper presents a closed form expression for the stochastic grinding force as a function of the grinding conditions and grit distribution. The stochastic grit density function is introduced to describe the random grit distribution of the rotating wheel. The dynamic grinding force is formulated as the convolution of a single-grit force and the grit density function. The single-grit force is obtained from analysis of the grinding geometry and treated as a deterministic impulse response of the grinding process. The spectrum characteristics of the grinding force are investigated in the frequency domain, where the power spectrum density (PSD) of the total grinding force can be expressed as a product of the energy spectrum density of the single-grit force and the PSD of the grit density function. The analytical nature of the PSD expression of the grinding force allows the identification of the PSD of the grit density function and the mechanistic grinding coefficients, and facilitates the analysis of the effects of the grit distribution and grinding conditions upon the grinding force. A series of grinding experiments were performed and their results discussed to validate this model. The results show that predictions drawn from the theoretical model are substantiated by the PSD, variance, and time domain signal of the experimentally measured grinding forces under various grinding conditions.

1. Introduction

As a ground surface is generated by the composite trajectory of all grits engaging with the work, its quality is greatly affected by the wheel and machine vibration generated in the process [1–4]. There are two main sources leading to grinding vibration [5]: self-excited chatter and forced vibration. Self-excited chatter is generated by an unstable grinding condition. Forced vibration results from forcing sources, which include the dynamic grinding force and unbalanced wheel rotation. Most causes of vibration have been reported [5–8] but none are related to the dynamic force generated by the congregated impulsive action of cutting grits. Unlike the well-defined tool geometry in a turning or milling process, the random nature of the grit distribution on a wheel surface makes it difficult to derive an analytical force model describing the dynamic force behavior of a grinding process. As the surface roughness requirement of a ground surface reaches the order of nanometers in ultraprecision machining, the effect of the dynamic component in a grinding force becomes significant and a force model considering the dynamic component in a grinding process is therefore helpful in the planning of process variables and in the design of grinding wheel and machine.

Most research regarding grinding forces begins with an analysis of the grinding geometry [9–12] and mechanism [13–18] in single-grit behavior. The total grinding force is then derived by superimposing the forces of the grits acting during a process. These models deal with the average grinding force based on a uniform grit distribution or an estimated average grit density on the wheel without considering the random nature of the grinding process. The random distribution of the grits on the wheel surface, however, makes the grinding process stochastic in nature. To account for the random distribution of the grits, both stochastic and probability theories have been widely applied in research regarding grinding processes [19–23]. Most of these studies employ stochastic descriptions such as the average value, standard deviation, or power spectrum density (PSD) to characterize the morphology of the wheel and ground surface [19,20]. Some researchers have simulated a ground surface morphology and estimated the grinding force using statistical analysis of the ground chip geometry [22,23]. Only the average grinding force has been considered in these researches, however, and the dynamic grinding force has not yet been investigated. The dynamic component of the grinding force, as in a milling process, originates from the variation of the chip load, exit and entry angle, and direction in a single grit as well as the sequential, intermittent engagement of all grits on a wheel. Therefore, this paper includes a grinding geometry analysis, stochastic theory, as well as a systematic method to present a grinding force model, including both the average and dynamic components.

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doi:10.1016/j.ijmachtools.2008.05.012

Nomenclature

b	uncut chip width
C	average number of grits per unit area
D	wheel diameter
d_a	grinding width
d_r	grinding depth
ESD	energy spectrum density
$\mathbf{f}$, f_x, f_y	total cutting force vector and its components in Cartesian coordinates
$\mathbf{f}_g$, f_{gt}, f_{gr}, f_{gx}, f_{gy}	cutting force vector of a single grit and its components, respectively, in tangential–radial and Cartesian coordinates
F_x, F_y	Fourier transform of the total cutting force components
gd	grit density function
k_t, k_r	grinding coefficients: specific grinding energy and the ratio of the radial to the tangential grinding force
PSD	power spectrum density
p_1, p_2	normalized tangential grinding force per unit chip load in the X and Y directions, respectively
p_x, p_y	elementary cutting function in the X and Y directions, respectively
P_1, P_2	Fourier transform of p_1 and p_2, respectively
P_x, P_y	Fourier transform of p_x and p_y, respectively
r	ratio of the chip width to thickness
R	radial position of cutting point on the wheel
S_{f_x}, S_{f_y}	power spectrum density of the grinding force in the X and Y directions, respectively
S_{gd}	power spectrum density of the grit density function
S_{p_x}, S_{p_y}	power spectrum density of the elementary cutting function in the X and Y directions, respectively
t_x, t_c	feed per grit and uncut chip thickness, respectively
V_S, V_W	wheel speed and work velocity, respectively
w	cutting window function
β	angular position of cutting point on the wheel
ϕ	wheel rotational angle
η_r	d_r/D
μ_{f_x}, μ_{f_y}	average grinding force in the X and Y directions, respectively
μ_{gd}	average grit density
θ, θ_1, θ_2, θ_r	angular position of the cutting point in the work coordinate, entry angle, exit angle, and active range of grinding angle, respectively
ω	normalized frequency
ω_0	spindle speed

The basic mechanism and behavior in grinding are similar to milling processes [16]. Therefore, this paper refers to a convolution approach that Wang et al. [24] and Wang and Zheng [25] applied to establish an analytical milling force model. In these works, the milling force of a single flute was treated as an impulse response of the milling force system and flute sequence of an end mill as the system's input. The output of this system as the total milling force is the convolution integral of the flute sequence function and the single-flute force response. Although the grits on a wheel play the same role as the flutes on a milling cutter, their distribution is not as regular and varies during the process. Based on the stochastic nature of grits in the time and spatial domain, a random function is introduced to describe the random grit distribution in this paper. The PSD analysis, which has been widely used to inspect the frequency characteristics of random processes, is then applied. The PSD of the total grinding force can be derived as an analytical expression of grinding conditions and active grit distribution of the wheel. Thus, the force under different grinding conditions can be predicted and the effects of grinding conditions and wheel constitution can be analyzed.

In the following, the analysis of the grinding geometry in a single grit is first performed to develop its mechanistic force expression. A stochastic grit density function is then introduced to describe the distribution of grits on a wheel surface and applied to obtain the total dynamic forces. Following the PSD analysis of the force model, the formula and procedure to identify the grinding coefficients and PSD of the grit density function are then introduced. Finally, a series of experiments are conducted to verify the model and discussions are made to illustrate the effects of various grinding conditions.

2. The establishment of a stochastic grinding force model

2.1. Force expression for a single grit

The position of an engaging active grit on the wheel and its position relative to the work are defined within the cylindrical wheel coordinate (R, β) and work coordinate (X, Y), shown in Fig. 1(a). The tangential grinding force for a single grit can be expressed as the product of a specific grinding energy and a chip load, which varies continuously with the cutting position. According to the grinding geometry for a triangular cross-sectional chip [11] as illustrated in Fig. 1(b), the instantaneous single-grit force can be expressed as

$$f_{gt}(\theta) = k_t A_c(\theta) = \tfrac{1}{2} k_t b(\theta) t_c(\theta) \tag{1}$$

where θ is the angular position of the cutting point in the work, as shown in Fig. 1(a). k_t is the specific grinding energy (J/mm^3), and $b(\theta)$ and $t_c(\theta)$ are the instantaneous uncut chip width and thickness, respectively. Assuming the ratio of $b(\theta)$ and $t_c(\theta)$ to be a constant, r [11], (1) can be rewritten as

$$f_{gt}(\theta) = \tfrac{1}{2} k_t r t_c^2(\theta) \tag{2}$$

The uncut chip thickness $t_c(\theta)$ uses the same common expression as in milling [26] and can be expressed as

$$t_c(\theta) = t_x \sin(\theta), \quad 0 \leqslant \theta \leqslant \pi \tag{3}$$

where t_x is the average feed per single grit and can be obtained from the expressions of the maximum chip thickness for a common grinding process [11]:

$$t_{c\,max} = 2 t_x \eta_r^{0.5} = \left[\frac{6 V_W \eta_r^{0.5}}{C r V_S}\right]^{0.5} \tag{4}$$

In this equation, V_W is the feed of the wheel, V_S is the wheel peripheral speed and η_r is the ratio of the grinding depth d_r to the wheel diameter D. The average number of grits per unit area C can be determined by

$$C = \frac{6 V_g}{(\pi d_g^2)(1 - f_b^{1/3})} \tag{5}$$

where V_g is the volume percentage of grits in a wheel, d_g the average grit diameter, and f_b the fraction bond fracture which is related to the wheel grade [11]. By rearranging (4), the feed per tooth can be found to be

$$t_x = \left(\frac{3 V_W}{2 C r V_S \eta_r^{0.5}}\right)^{0.5} \tag{6}$$