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An analytical approach to dynamic spalling of brittle materials

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A B S T R A C T

An analytical method for the spalling phenomenon of brittle materials is presented, based on the analytical expression, for the dynamic tensile strength of brittle materials with respect to the local strain rate, proposed by us before. The numerical results are in good agreement with previous experimental data and computational results by using the FEM method. Moreover, it also provides an indirect verification for the viewpoint that the dynamic tensile load-carrying capacity of brittle materials can be determined by the quasistatic material parameters and the boundary loading, or, in other words, the so-called strain-rate effect on the material strength should not be considered as an intrinsic material property anymore, as argued recently by the authors.

1. Introduction

Spalling fracture is one of the dynamic failure patterns of materials resulted from the tensile wave generated by reflection of an incident compressive wave at a free surface. It is of both practical and scientific importance for the design of military or civil protective facilities to resist the intensive dynamic loads such as that resulted from explosion, high-velocity impact and energy deposition. Over the past decades, much research work on the spalling phenomenon [1–6] has been opened. For example, Brara, Klepaczko [6,7] developed an experimental arrangement to measure the first spalling position of a bar under impact loadings; recently, taking the damage accumulation into account, Davison and Stevens [8] proposed a quantitative/predictive model to predicate the spalling fracture, and Taylor et al. [9] developed a microcrack-induced damage accumulation model to investigate the dynamic fracture behavior of brittle rock. However, as is well known, it is very difficult to carry out an analytical investigation into the spalling issue involving the so-called strain-rate effect on the dynamic strength of materials, and hence most of the previous research work on the spalling phenomenon can only be done experimentally or numerically, which limited apparently the profound understanding about such a crucial issue. Recently, some researchers [10–12] have developed an important idea that the dynamic tensile strength should not be consider as an intrinsic property of materials. They also indicated the dynamic tensile load-carrying capacity of brittle materials could be determined by the static material properties as well as the interaction between the material properties and the exterior or boundary loadings. Furthermore, we [13] have obtained an analytical expression for the dynamic strength of brittle materials, and the corresponding numerical results are in good agreement with the opened experimental data. Therefore, on the one hand, our research provides a verification of the above mentioned idea and, on the other hand, lays the theoretical foundation of the further analytical studies on the dynamic failure of brittle materials.

This research is a succession of our previous work. The objective of this study is to investigate analytically the one-dimensional spalling phenomenon under dynamic loadings *via* the analytical expression, obtained by the authors of this study before, and then deduce the analytical expression for the spalling thickness. Moreover, to verify the availability of the derived formula, the corresponding numerical results are presented and compared with the previous experimental data as well as the numerical results computed by using the FEM method. In this way, the analytical expression can prepare the ground for the further deep analytical investigation into the spalling phenomenon, and it also provides an indirect verification of the above mentioned idea on the strain-rate effect on the dynamic strength of materials. Moreover, it is worthy to be pointed out that the analytical expression implies that the spalling thickness can be determined directly only by the static material parameters, without considering any dynamic material parameter. On the other hand, many previous models or solutions [4,9,14,15] for spalling are based on the parameters obtained by

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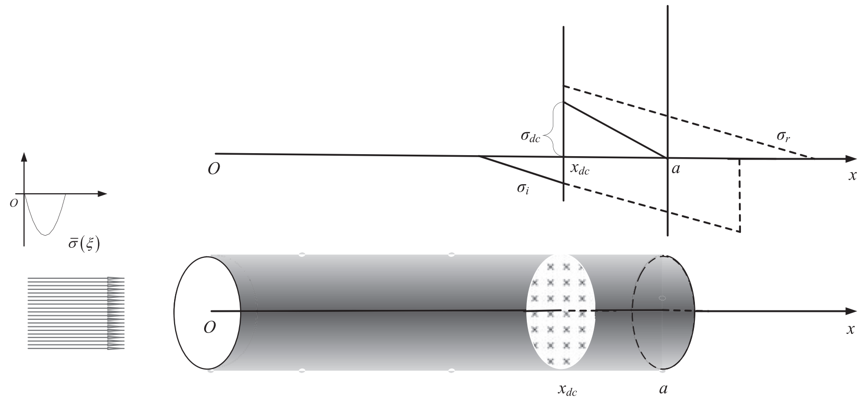

Fig. 1. One-dimensional stress plane longitudinal wave.

dynamic experiment. Thus the analytical solution of this research is of important significance especially in practical engineering applications.

This study starts with the brief introduction. Then, in Section 2, the one-dimensional initial boundary problem for the spalling phenomenon, based on the analytical expression of the dynamic strength, is presented and solved analytically to deduce the analytical expressions for spalling. In Section 3, in order to verify the availability the analytical expression, a numerical solution is derived by using the structural-temporal criterion directly; moreover, the numerical results corresponding to the both calculation methods are compared with opened experimental data and previous FEM methods, together with some further discussions. Finally, several conclusions are drawn out in Section 4.

2. Theoretical model and solution

A one-dimensional stress plane longitudinal wave model in a bar of length a is used in this study as illustrated in Fig. 1. Under the dynamic stress boundary loading $\sigma(0, t)$ on the left end of the bar, a one-dimensional stress plane longitudinal wave begins instantaneously at $t = 0$ to propagate in the direction of the positive x axis. After the incident compressive wave arrives at the right free end, it will be reflected and propagate backward as a tensile wave, and then the stress distribution is determined by the superposition of the incident compressive wave and the reflected tensile wave. The spalling will occur at a certain location x_{dc} in the bar, when at which the superposed tensile stress satisfies a given fracture criterion. For such a dynamic problem, the governing differential equation can be written as

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c_L^2}\frac{\partial^2 u}{\partial t^2} \tag{1}$$

where $u = u(x, t)$ is the particle displacement; $c_L = (E/\rho)^{1/2}$ is the propagating velocity of the plane longitudinal wave in the material under consideration, and ρ and E are the mass density and Young's modulus, respectively. For the bar initially stress-free and at rest, the initial conditions are

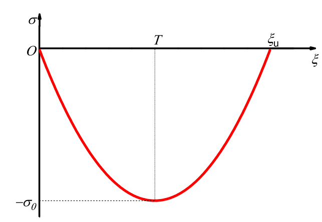

Fig. 2. Boundary loading.

$$u(x,0) = 0,\ \ v(x,0) = \frac{\partial u(x,0)}{\partial t} = 0 \tag{2}$$

where v is the particle velocity. Without loss of generality, a quadratic compressive stress waveform can be assumed as the boundary loading as depicted in Fig. 2, which can be written in the following form

$$\overline{\sigma}(\xi) \equiv \sigma(0,\xi) = kT^2 - k(\xi - T)^2 \tag{3}$$

where $T > 0$ is the half-cycle of the boundary loading, ξ is the time variable on the boundary $x = 0$ and $k < 0$ is a constant for the given boundary loading. In addition, for convenience, the absolute value of the amplitude of the boundary load is denote as σ_0, and it is easy to verify from Eq. (3) that $\sigma_0 = -kT^2$. Thus, Eqs. (1)–(3) compose of the initial-boundary value problem for the dynamic issue under consideration.

The general solution of the initial-boundary value problem Eqs. (1)–(3) can be derived as [16].

$$u(x,t) = -2B\int_0^{t-x/c_L} \overline{\sigma}(\xi)d\xi \tag{4}$$

where $B = c_L/2E$ is a constant. Substituting the boundary loading Eq. (3) into the solution Eq. (4), after a tedious but direct deduction, the incident compressive wave $\sigma_i(x, t)$ in the bar can be obtained as following:

$$\sigma_i(x,t) \equiv \sigma\left(t - \frac{x}{c_L}\right) = \begin{cases} 0 & 0 \le x \le a,\quad t < \dfrac{x}{c_L} \\ kT^2 - k\left(t - \dfrac{x}{c_L} - T\right)^2 & 0 \le x \le a,\quad \dfrac{x}{c_L} \le t < \dfrac{x}{c_L} + 2T \end{cases} \tag{5}$$