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Analysis of transitions between different ringing schemes of the church bell

P. Brzeski [a], T. Kapitaniak [a], P. Perlikowski [a, b, *]

[a] Division of Dynamics, Lodz University of Technology, Stefanowskiego 1/15, 90-924 Lodz, Poland
[b] Department of Civil & Environmental Engineering, National University of Singapore, 1 Engineering Drive 2, Singapore 117576, Singapore

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ABSTRACT

In this paper we investigate dynamics of church bells, characterize their most common working regimes and investigate how to obtain them. To simulate the behavior of the yoke-bell-clapper system we use experimentally validated hybrid dynamical model developed basing on the detailed measurements of the biggest bell in the Cathedral Basilica of St Stanislaus Kostka, Lodz, Poland. We introduce two parameters that describes the yoke design and the propulsion mechanism and analyze their influence on the systems' dynamics. We develop two-parameter diagrams that allow to asses conditions that ensures proper and smooth operation of the bell. Similar charts can be calculated for any existing or non-existing bell and used when designing its mounting and propulsion. Moreover, we propose simple and universal launching procedure that allows to decrease the time that is needed to reach given attractor. Presented results are robust and indicate methods to increase the chance that the instrument will operate properly and reliably regardless of changes in working conditions.

1. Introduction

Bells are one of the oldest musical instruments which still play an important cultural role. They were invented in China but today their sound announces major events all around the world. Depending on the region bells are mounted in a number of different ways basing on local customs and tradition. In Europe we can encounter three characteristic mounting layouts: Central European, English and Spanish. In Central Europe bells usually tilt on their axis with maximum amplitude of oscillations below 90°. In the English system the amplitude of oscillations is greater and bells perform nearly a complete rotations in both directions. Conversely, in the Spanish system bells rotate continuously in the same direction. All these mounting layouts were developed throughout centuries basing on experience and intuition of bell-founders and craftsmen. It is common that the bells are cast using casting mould passed down for ages from father to son and so forth. Although the design of a bell, its yoke, clapper and a belfry has been being improved for ages, proper modeling of their dynamics is still a challenging task.

The dynamics a yoke-bell-clapper system is extremely complex and difficult to analyze due to nonlinear characteristic, repetitive impacts and complicated excitation. Moreover, depending on the mounting layout different dynamical states can be observed and each type of yoke has its own specific properties. However, already in 19th century Wilhelm Veltmann tried to describe mathematically the behavior of the famous Emperor's bell in the Cologne Cathedral [17,18]. He used simplified equations of motion and explained why the clapper always remained on the middle axis of the bell instead of striking its shell. His model was developed basing on the equations of a double physical pendulum. Heyman and Threlfrall [5] use similar model to estimate inertia forces induced by a swinging bell. The knowledge of loads induced by ringing bells is crucial during the design and restoration processes of belfries. That is why there is a number of works considering dynamic interactions between bell towers and bells mounted in different manners. Muller [12], Steiner [16] and Schutz [14] focus on Central European mounting system which is also considered in the German DIN standard [1]. We can find similar studies concerning the Spanish system [2,3] and the English system [6,7]. Ivorra et al. present the comparison between the three mounting layouts and prove that in the Spanish system forces transmitted to the supporting structure are significantly lower than in the other two. Results presented in Refs. [4,10] show that in many cases we can

* Corresponding author. Division of Dynamics, Lodz University of Technology, Stefanowskiego 1/15, 90-924 Lodz, Poland.
E-mail address: przemyslaw.perlikowski@p.lodz.pl (P. Perlikowski).

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improve bells' working conditions slightly modifying the yoke or its support.

Apart from the studies concerning interactions between bells and their supports there is a number of publications focusing on the dynamics of the instrument itself or the clapper to the bell collisions. Klemenc et al. contributed with a series of papers [8,9] devoted to the analysis of the clapper-to-bell impacts. Authors investigate the consequences of the repetitive hits and compare experimental data with numerical results obtained from the finite-element model. Presented results prove that full-scale finite-element model is able to reproduce the effect of collisions but requires long computational times and complex, detailed models. Therefore, it would be difficult to use such models to analyze the dynamics of bells. Because of that, recently we observe the tendency to use hybrid dynamical models which are much simpler and give accurate results with less modeling and computational effort. In Ref. [11] authors propose lumped parameter model of the bells mounted in Central European system and prove that with the model we are able to predict impact acceleration and bell's period of motion.

In our previous publication [13] we present an improved hybrid dynamical model of the yoke-bell-clapper system. All parameter values involved in the model have been determined basing on the measurements of the biggest bell in the Cathedral Basilica of St Stanislaus Kostka, Lodz, Poland. Proposed model is validated by comparing the results of numerical simulations with experimental data. The presented results show that the introduced model is a reliable predictive tool and can be used for further case studies.

In this paper we describe the most common working regimes of bells and investigate how the yoke design and the propulsion mechanism influence their dynamics. To simulate the behavior of the yoke-bell-clapper system we use the hybrid dynamical model that we present in detail in our previous publication [13]. We characterize the solutions that can be considered as the proper operation of the bell and analyze how the geometry of the yoke and the driving motor output affect the dynamics of the system. We introduce two influencing parameters and develop diagrams that allow to asses how the bell's behavior depends on the yoke type and propulsion amplitude. Such plots describe how presumed working regimes can be obtained and can be beneficial during the design and/or restoration processes of the bells. In addition, we investigate the time that is needed to reach given attractor. Presented results prove that in some cases special launching procedure of the instrument should be introduced to shorten the time of transient motion. Finally, we propose simple and universal control of the driving mechanism that allows to decrease transient time significantly.

The paper is organized as follows. In Section 2 we describe the hybrid dynamical model of the church bell and introduce parameters that influence the system's dynamics. In Section 3 we characterize the 7 most common working regimes and in Section 4 investigate how they can be obtained. Finally, in Section 5 the conclusions are drawn.

2. Model of the system

The hybrid dynamical model of the yoke-bell-clapper system that we consider in this paper has been described in detail in our previous publication [13]. To develop the model and determine its parameters values we have performed detailed measurements of the existing bell named *"The Heart of Lodz"* (the biggest bell in the Cathedral Basilica of St Stanislaus Kostka in Lodz). Using the same bell we have tuned and validated the model by comparing the results of numerical simulations with the data collected during a series of experiments. Presented results show that the model is a reliable predictive tool which can be used to simulate the behavior of a wide range of yoke-bell-clapper systems. In next subsections we briefly describe the derivation of the equations of motion and present the influencing parameters.

2.1. Geometry of the yoke-bell-clapper system

The model that we use is build up based on the analogy between freely swinging bell and the motion of the equivalent double physical pendulum. The first pendulum has fixed axis of rotation and models the yoke together with the bell that is mounted on it. The second pendulum is attached to the first one and imitates the clapper. The photo of the bell that has been measured to obtain the parameters values is presented in Fig. 1 (a). In Fig. 1 (b, c) we show schematic model of the bell indicating the position of the rotation axes of the bell o_1, the clapper o_2 and presenting parameters involved in the model. For simplicity, henceforth, we use term "bell" with respect to the combination of the bell and it's yoke which we treat as one solid element.

The model involves eight physical parameters. Parameter L describes the distance between the rotation axis of the bell and its center of gravity (point C_b), l is the distance between the rotation axis of the clapper and its center of gravity (point C_c). The distance between the bell's and the clapper's axes of rotation is given by parameter l_c. The mass of the bell is described by parameter M, while parameter B_b characterizes the bell's moment of inertia referred to its axis of rotation. Similarly, parameter m describes the mass of the clapper and B_c stands for the clapper's moment of inertia referred to its axis of rotation.

Considered model has two degrees of freedom. In Fig. 1 (d) we present two generalized coordinates that we use to describe the state of the system: the angle between the bell's axis and the downward vertical is given by φ_1 and the angle between the clapper's axis and downward vertical by φ_2. Parameter α (see (1) (d)) is used to describe the clapper to the bell impact condition which is as follows:

$$|\varphi_1 - \varphi_2| = \alpha \tag{1}$$

Synonymously, collision between the bell and the clapper occurs when the absolute difference between the bell's and the clapper's angular displacements is equal to α.

2.2. Equations of motion – modeling of an oscillatory motion of the system

In this section we present the mathematical model that we use to simulate oscillatory motion of the investigated yoke-bell-clapper system. We use Lagrange equations of the second type and derive two coupled second order ODEs that describe the motion of the considered system (full derivation can be found in Ref. [13]):

$$\left(B_b + ml_c^2\right)\ddot{\varphi}_1 + ml_cl\ddot{\varphi}_2\cos(\varphi_2 - \varphi_1) - ml_cl\dot{\varphi}_2^2\sin(\varphi_2 - \varphi_1) + (ML + ml_c)g\sin\varphi_1 + D_b\dot{\varphi}_1 - D_c(\dot{\varphi}_2 - \dot{\varphi}_1) = M_t(\varphi_1), \tag{2}$$

$$B_c\ddot{\varphi}_2 + ml_cl\ddot{\varphi}_1\cos(\varphi_2 - \varphi_1) + ml_cl\dot{\varphi}_1^2\sin(\varphi_2 - \varphi_1) + mgl\sin\varphi_2 + D_c(\dot{\varphi}_2 - \dot{\varphi}_1) = 0. \tag{3}$$

where g stands for gravity and $M_t(\varphi_1)$ describes the effects of the linear motor propulsion. The motor is active – and excites the bell – when its deflection from vertical position is smaller than $\pi/15$ [*rad*]