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Beyond classical dynamic structural plasticity using mesh-free modelling techniques

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ABSTRACT

The problem of modelling the transient response of an elastic-perfectly-plastic cantilever beam, carrying an impulsively loaded tip mass, is often referred to as the Parkes cantilever problem [25]; The permanent deformation of a cantilever struck transversely at its tip, Proc. R. Soc. A., 288, pp. 462). This paradigm for classical modelling of projectile impact on structures is re-visited and updated using the mesh-free method, smoothed particle hydrodynamics (SPH). The purpose of this study is to investigate further the behaviour of cantilever beams subjected to projectile impact at its tip, by considering especially physically real effects such as plastic shearing close to the projectile, shear deformation, and the variation of the shear strain along the length and across the thickness of the beam. Finally, going beyond macroscopic structural plasticity, a strategy to incorporate physical discontinuity (due to crack formation) in SPH discretization is discussed and explored in the context of tip-severance of the cantilever beam. Consequently, the proposed scheme illustrates the potency for a more refined treatment of penetration mechanics, paramount in the exploration of structural response under ballistic loading. The objective is to contribute to formulating a computational modelling framework within which transient dynamic plasticity and even penetration/failure phenomena for a range of materials, structures and impact conditions can be explored *ab initio*, this being essential for arriving at suitable tools for the design of armour systems.

1. Introduction

The response of a tip-loaded metal cantilever beam is a classical problem in elasto-plastic structural dynamics and has been the subject of interest for many years to both experimentalists and modellers.[1] This problem is frequently referred to as the '*Parkes cantilever problem*' in view of the classical rigid-plastic, small-deflection analysis and experimental results due to [25]. It has been the subject of long-term interest primarily because it introduces phenomena and structural concepts associated with important topics of dynamic structural plasticity (e.g. the notion of travelling plastic hinges) and, in particular, to ballistic impact.

Many analytical, numerical and experimental studies have been reported (*vide* the books by Refs. [10,13,14,34] etc.). The Parkes problem is a prototype for many structural impact problems, in which elastic–plastic transient behaviour plays a leading role. The modelling of a cantilever beam, subjected to an impulse load at its tip, provides a key introduction to such features as travelling plastic hinges and the role of elasticity in an otherwise plasticity-dominated scenario [10]. Additionally, in ballistics, the interaction between a projectile and a structure can lead to penetration and perforation of the target, introducing fracture as another failure mechanism.[2]

Symonds and Fleming [37] revisited the Parkes problem to extend the modelling to elastic–plastic large deflection scenarios, but mainly to justify their use of the mode technique. In order to understand the role of elasticity in structures undergoing impact loading [30], examined the transient bending-only behaviour of an elastic-perfectly plastic cantilever beam carrying a tip mass which is subjected to impulsive loading. Both Symonds and Fleming and Reid and Gui used an ABAQUS-FEM model. Reid and Gui showed

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[1] This paper is dedicated to the memory of Professor W. Johnson FRS, FREng, who died on 13th June 2010, and who introduced the second author to the 'Parkes' problem during the writing of his book 'Impact Strength of Materials' in 1983.

[2] The important issue of the effect of projectile deformation on the final failure patterns and fracture threshold is not considered here and will be dealt with in a future communication.

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that, during the initial stage of the deformation process, a plastic region (hinge) forms near the tip and propagates towards the root. However, due to the reflected precursor elastic bending wave, the progress of the plastic hinge is arrested at approximately the centre of the cantilever, giving the beam a characteristic local ‘kink’ there (Fig. 1). Thus the modal phase [37] of the beam motion further gets delayed along with the root rotation.

One of the major limitations of the FEM in dealing with such problems is that, close to the projectile or wherever the beam experiences large, local deformations with sharp gradient layers, element distortion might affect the accuracy of computations. In the last few decades attempts have been made to develop new computational techniques, e.g. mesh-free methods such as Smoothed Particle Hydrodynamics (SPH) that provide freedom from element-based domain discretization and hence, to an extent, from the difficulties associated with element distortion. This paper, therefore, attempts to update the computational approach of modelling this classical problem.

SPH [9,18,20,23] is a mesh-free tool that has now found considerable appeal among researchers for the numerical modelling of problems in structural impact mechanics. This is particularly so in problems where the structure with limited material strength fails at several locations causing breakage and violating the continuum nature of the post-deformed macrostructure. In SPH, a continuum is modelled as a set of discrete particles. The description of a locally smooth deformation field is then created using a moving, compactly-supported kernel function. This allows the particles to interact with each other such that the local conservation equations around every particle are satisfied in a weighted averaged sense. SPH has been used successfully in fluid dynamics and astrophysical problems [24] since its inception. However its adaptation into dynamic structural plasticity is relatively new [16,17] and poses a major challenge to computational/numerical analysts. In addition to the application of SPH to a wide range of problem related to impact mechanics [3,5,15,19], over the years its numerical aspects have been gradually improved, some inherent drawbacks have been identified and corrective measures have been proposed [11,29,31,33,35,36]. A comprehensive survey on SPH may also be found in the review paper by Ref. [22] and the references therein.

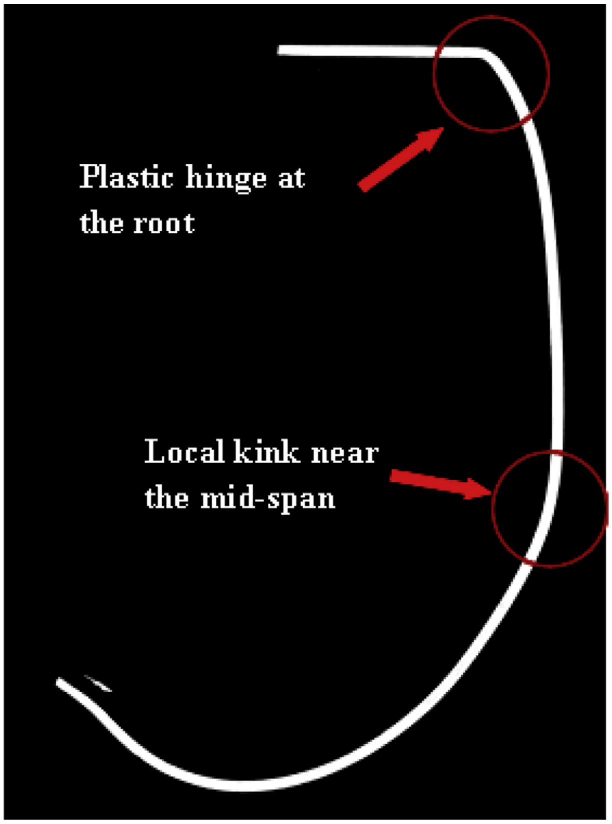

Fig. 1. Final deformed shape of an aluminium cantilever beam after impact at its tip [from Ref. [12]].

The main objective of this study is to investigate further the Parkes problem using the SPH approach. Specifically, the aim is to improve upon the earlier numerical work by Ref. [30]. Since SPH-based discretization of the continuum is not constrained by dimensionally-reduced structural theories (such as the Euler-Bernoulli assumption), physically real effects such as plastic shearing under the projectile, shear deformation across the thickness can also be tested through these methods. Finally a strategy to incorporate failure (through crack initiation and propagation leading to breakage) in SPH computation is discussed and explored in modelling ‘tip’ severance and getting towards a treatment of the notion of structure/projectile ballistic limits.

2. SPH method: a brief account

For a better appreciation of the development to follow, the basic steps involved in SPH applied to solid mechanics problems are briefly outlined in this section.

2.1. Governing equations

For generality, the governing equations are expressed within the continuum mechanics framework. The conservation equations are given by

$$\frac{d\rho}{dt} = -\rho\frac{\partial v^{\beta}}{\partial x^{\beta}} \tag{1}$$

$$\frac{dv^{\alpha}}{dt} = \frac{1}{\rho}\frac{\partial \sigma^{\alpha\beta}}{\partial x^{\beta}} \tag{2}$$

$$\frac{de}{dt} = \frac{\sigma^{\alpha\beta}}{\rho}\frac{\partial v^{\alpha}}{\partial x^{\beta}} \tag{3}$$

$$\frac{du^{\alpha}}{dt} = v^{\alpha} \tag{4}$$

where, for at any material point, ρ denotes the mass density, e is the specific internal energy, u^{α}, v^{α} and $\sigma^{\alpha\beta}$ are respectively elements of the displacement vector, velocity vector and Cauchy stress tensor, x^{α} is the current spatial coordinate, d/dt is the time derivative taken in the moving Lagrangian frame and the superscripts $\alpha, \beta = 1, 2, 3$ are integer indices for the three spatial directions. The stress components $\sigma^{\alpha\beta}$ may be written in terms of hydrostatic and deviatoric parts as:

$$\sigma^{\alpha\beta} = -P\delta^{\alpha\beta} + S^{\alpha\beta} \tag{5}$$

where P and $S^{\alpha\beta}$ are respectively the scalar pressure and components of the traceless, symmetric deviatoric stress tensor with $\delta^{\alpha\beta}$ being the Kronecker delta.

Now, as noted from Equations (1)–(4), changes in the position vector are mainly governed by the stresses induced at that particular position. Any loading, here in the form of an impulse, would typically generate elastic (and plastic) stress waves that manifest themselves in the global response pattern of the system. Potentially, the magnitude of the induced stress wave may cause strength failure of the material. In order to estimate the stress field over the