International Journal of Impact Engineering 37 (2010) 723–734

Contents lists available at ScienceDirect

International Journal of Impact Engineering

journal homepage: www.elsevier.com/locate/ijimpeng

# An approach to robust optimization of impact problems using random samples and meta-modelling

David Lönn [a,*], Ørjan Fyllingen [b], Larsgunnar Nilssona [a]

[a] Division of Solid Mechanics, Linköping University, SE-58183 Linköping, Sweden
[b] Structural Impact Laboratory (SIMLab), Department of Structural Engineering, Norwegian University of Science and Technology, NO-7491 Trondheim, Norway

ARTICLE INFO

*Article history:*
Received 28 November 2008
Received in revised form
2 July 2009
Accepted 3 July 2009
Available online 10 July 2009

*Keywords:*
Robust optimization
Geometric imperfections
Meta-model
Robustness

ABSTRACT

Conventionally optimized structures may show a tendency to be sensitive to variations, for instance in geometry and loading conditions. To avoid this, research has been carried out in the field of robust optimization where variations are taken into account in the optimization process. The overall objective is to create solutions that are optimal both in the sense of mean performance and minimum variability. This work presents an alternative approach to robust optimization, where the robustness of each design is assessed through multiple sampling of the stochastic variables at each design point. Meta-models for the robust optimization are created for both the mean value and the standard deviation of the response. Furthermore, the method is demonstrated on an analytical example and an example of an aluminium extrusion with quadratic cross-section subjected to axial crushing. It works well for the chosen examples and it is concluded that the method is especially well suited for problems with a large number of random variables, since the computational cost is essentially independent of the number of random variables. In addition, the presented approach makes it possible to take into consideration variations that cannot be described with a variable. This is demonstrated in this work by random geometrical perturbations described with the use of Gaussian random fields.



## 1. Introduction

To study the axial buckling of a straight profile is challenging. Small perturbations of the design or loading conditions may create large variations in structural responses, as seen in for instance Fyllingen et al. [1]. Structures that strongly change their behaviour when subjected to small variations are not robust, and this property is rarely sought. Traditional deterministic optimization methods applied to structures subjected to impact loading may also create non-robust designs. This problem can be resolved by accounting for variations in the design variables and environmental conditions when performing the optimization, i.e. performing a robust optimization. This rather fast growing research area is summarised by e.g. Beyer and Sendhoff [2].

The concept of robust design is to find an optimal design, which is not only optimal in the sense of mean response value, but also has a minimal variation of the response when subjected to stochastic variations. However, there are several ways to account for variations, i.e. different ways of including the variations in the optimization formulation. Several papers have included uncertainties in the constraints by changing the constraints from being always fulfilled to being fulfilled to some probability chosen. These methods with probabilistic inequality constraints are denoted as Reliability-Based Design Optimization approaches (RBDO), see e.g. Gu et al. [3]. According to for instance Beyer and Sendhoff [2], there is no consensus in the literature as to whether RBDO should be regarded as robust optimization or not, since the formulation does not imply a minimisation of the response variations, but rather an inclusion of a safety margin in the constraints. In order to explicitly minimise the variations of a response, this entity must either be present in the objective function, e.g. as in Doltsinis et al. [4], or be given an upper limit in a constraint so that the variations in the response is minimised in order to satisfy the constraint.

The grand challenge lies in evaluating the responses and their variations for computationally costly applications. Responses from impact loading conditions generally require long computing times and it is common to use meta-models for approximating the responses. The meta-models are built from carefully selected response evaluations, where the chosen sets of variable values are denoted as the Design Of Experiments (DOE). Using the meta-models, it is then possible to get an approximation of the non-evaluated designs. Several papers extend the usage of the

* Corresponding author. Tel.: +46 13 282475; fax: +46 13 282717.
*E-mail address:* david.lonn@liu.se (D. Lönn).

0734-743X/$ – see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijimpeng.2009.07.002

constructed meta-models to making approximate evaluations of response dispersion data, typically Monte Carlo analyses performed on the meta-models, e.g. Lee et al. [5], Sinha et al. [6] and Ait Brik et al. [7]. In order for this methodology to be accurate, the meta-model must also be accurate. For a complete assessment of the dispersions, one must also consider the uncertainties in the meta-model itself, e.g. as in the work of Martin and Simpson [8].

Inspired by the work of Vining et al. [9,10], several other recent papers use the dual response surface approach in which two response surfaces are created, one for the mean and one for the variance or standard deviation of a response. In contrast to meta-model based Monte Carlo analyses, e.g. Kovach and Cho [11] and Shin and Cho [12] use replicates of the same design in order to obtain estimates of the variances for the different responses.

Our work presents an alternative approach to robust optimization. Two meta-models are used, one for the mean and one for the standard deviation of the response. Random (uncontrollable) variables are not included in the variable space. Instead, in each design point, an assessment of the mean and standard deviation of a response is made based on a predetermined set of random samples for the stochastic variables, i.e. the random variables and the design variables that are non-deterministic. The mean and the standard deviation of the response are approximated over the design variable space using Artificial Neural Network (ANN) meta-models. Standard optimization strategies implemented in the software LS-OPT are then applied to the robust optimization formulation in order to find an optimal robust design. The novelty in this work is the combination of several factors; the robust optimization formulation, using the same random samples throughout the optimization, the calculation of robustness in each design, the use of random fields and the optimization procedure.

Two examples of the proposed robust optimization approach are provided, one analytical design example and one Finite Element (FE) example, where the objective is to find the robust optimal size and position of a buckling trigger on a square aluminium tube subjected to impact loading. For the latter, a similar problem set-up has been presented by e.g. Missoum [13,14], but the solution techniques are different, as he maximises the probability that the mean response is above a certain value, whereas in this work, the variance in each design is also studied and used as an objective in the optimization. Finally, it is concluded that the method proposed here is especially well suited for problems with a large number of random variables.

## 2. Objective

The objective of this work is twofold. First, an alternative approach to robust optimization is presented. The method is not necessarily restricted to usage in a structural impact context, but the chosen structural example serves as a good demonstrator. Advantages and restrictions of the proposed approach are presented in the discussion, as well as possible fields of application. Second, the approach is applied to the robust optimization problem of finding the optimal size and placement of a buckling trigger on an axially crushed aluminium profile. In this example, it is shown why a small trigger should be placed at one end of the axially crushed beam.

## 3. Theory

The following sections briefly present the theories that constitute the basis for this work.

### 3.1. Robust optimization

A robust optimization is an optimization where dispersions of the variables and responses are taken into account. Thus, the optimization problem can be formulated as a multiobjective problem with the minimum of the dispersion as an additional objective,

$$\min_x \left[\mu(f(x)), \sigma(f(x))\right] \tag{1}$$

with appropriate constraints. A common approach is to weight the two objectives linearly, i.e.

$$\min_x \bar{f}(x) = \alpha\frac{\mu(f(x))}{\mu_0(f(x_0))} + (1-\alpha)\frac{\sigma(f(x))}{\sigma_0(f(x_0))} \tag{2}$$

where the new objective function $\bar{f}$ is a linear combination of the mean, $\mu$, and the standard deviation, $\sigma$, of a stochastic response $f(x)$. By performing a normalisation, introducing $\mu_0$ and $\sigma_0$ denoting the mean and standard deviation of the initial designs response, the trade-off situation becomes independent of the size of the two terms in the objective function. The robustness of the optimal solution design will then only depend on the choice of the parameter $\alpha$. This is not required when the mean value and the standard deviation are of comparable magnitude.

The formulation above has been selected for this paper. However, true expressions for mean and standard deviation of responses are generally not known, and these entities must be replaced by approximations that are valid over the design domain. The robust optimization formulation changes to

$$\min_x \tilde{f}(x) = \alpha\frac{\tilde{\mu}(f(x))}{\mu_0(f(x_0))} + (1-\alpha)\frac{\tilde{\sigma}(f(x))}{\sigma_0(f(x_0))} \tag{3}$$

where $\tilde{\mu}$ and $\tilde{\sigma}$ are meta-model approximations of the true responses.

Since $\tilde{f}$ now is a smooth function, any standard optimization algorithm can be used for this problem. This work uses the Leapfrog Optimizer for Constrained Optimization (LFOPC) in conjunction with two different types of neural networks as meta-models, as implemented in LS-OPT, see Stander et al. [19].

### 3.2. Meta-modelling

Meta-models are constructed approximations of the responses over the design space. The approximations are built up from evaluations of response values for a carefully selected set of designs, denoted the Design of Experiments (DOE). Meta-models are most commonly used when each evaluation of the response is computationally costly and when a global or local approximation may increase the efficiency, e.g., in traditional optimization or sensitivity analysis. It is important to note that the robust optimization approach presented here is independent of the choice of meta-model. However, since every design point in this method requires several costly evaluations, it is wise to choose a meta-model where the results from every design evaluation are saved and reused, and where, consequently, the meta-model is refined for each iteration in the optimization procedure.

In this work, a meta-model based on a neural network which meets the above criteria is used. Moreover, according to e.g. Redhe [17], neural networks are also capable of capturing local changes of the response, such as a bifurcation in the buckling mode where the energy absorption changes rapidly with a small change in the design. This is not the case when a polynomial meta-model is used over the entire region of interest.

### 3.3. Artificial neural network

A neural network, or more precisely, an Artificial Neural Network (ANN), may be used to approximate complex relations