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Comparison of plasticity models for tantalum and a modification of the PTW model for wide ranges of strain, strain rate, and temperature

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ABSTRACT

Four well-known constitutive models for plastic deformation of materials, i.e., Johnson–Cook (JC), Zerilli–Armstrong (ZA), Voyiadjis and Abed (VA), and Preston–Tonks–Wallace (PTW), have been compared with reference to existing deformation data of tantalum in wide ranges of strain, strain rate, and temperature. All of these models reasonably describe the flow stress and the strain-hardening behavior only in the certain ranges of strain, strain rate, and temperature for which the models were developed. The PTW model with appropriate parameters most effectively describes the effects of strain rate and temperature in a wider range, except for strain hardening. The strain-hardening term of PTW was thus modified in the current work and the modified PTW demonstrated very good prediction for the constitutive behavior of tantalum in wide ranges of strain, strain rate, and temperature.

1. Introduction

With the merits of high density and high ductility, tantalum has been widely used as a liner material for explosively formed projectiles (EFPs). As tantalum is highly ductile, it can form a long aerostable projectile (EFP) out of a dish-shaped liner. Tantalum EFPs are efficient in penetrating a target due to the high velocity (1500–2000 m/s) and high density of tantalum (16.65 g/cm^3). Fig. 1 shows a schematic illustration of the EFP formation process and the final morphology of penetration into the target. For both the formation and penetration processes, strain, strain rate, and temperature change significantly. In order to ensure an accurate numerical simulation, therefore, a constitutive material model has to reflect the effects of strain, strain rate, and temperature in a reasonably wide range.

Thus far, indeed, numerous constitutive models have been proposed to describe plastic deformation. Some of them include, namely, the Steiberg–Guinan model (SG) [1], the Zerilli–Armstrong model (ZA) [2], the Johnson–Cook model (JC) [3], and the mechanical threshold stress model (MTS) [4], Voyiadjis and Abed (VA) model [5], and Preston–Tonks–Wallace (PTW) model [6]. Briefly introducing the proposed models, the SG model focused on a high-strain-rate deformation by ignoring deformation at low-strain rates. The ZA model was designed to account for initial dislocation density and dislocation moving mechanism. The JC and ZA models are known to describe the flow stress and strain-hardening behavior of material deforming at a low-strain rate and at temperature near room temperature. So-called mechanical threshold stress (MTS) model was introduced with the idea that plastic deformation is controlled by the thermally activated interactions of dislocations with obstacles [4]. The VA model aimed to improve the ZA model by modifying the evolution of mobile dislocation density, and it reportedly predicts the flow stress better at high-strain rates. In the PTW model, in addition to the thermally activated dislocation interactions that have dominant effect in a low-strain rate regime, the dislocation drag mechanism was introduced to cover strain rates over 10^7 s^{-1}.

There have been also many works regarding the experimental determination of the material parameters for the constitutive models. Briefly addressing some of them, Chen and Gray [7] determined the parameters of the constitutive relations of Ta and Ta–W alloys for the JC, ZA, and the MTS models, and also investigated the fitting characteristics of the measured flow stress in the range of strain from 0 to 0.8, strain rate from 0.001 s^{-1} to 5000 s^{-1}, and temperature from 25 °C to 1000 °C. In the case of Maudlin et al. [8,9], the MTS model parameters for DoD tantalum were determined by comparing the simulated anisotropic deformation of a tantalum rod with a piece-wise yield surface that was determined by an experimentally derived orientation distribution function. In order to describe the constitutive behavior using the MTS model, however, as many as seven parameters and three functions had to be decided.

Comparison of existing models is important for an accurate numerical simulation of the plastic deformation phenomenon [10]. Of many constitutive models, the current work has compared the prediction capability of four well-known plastic constitutive

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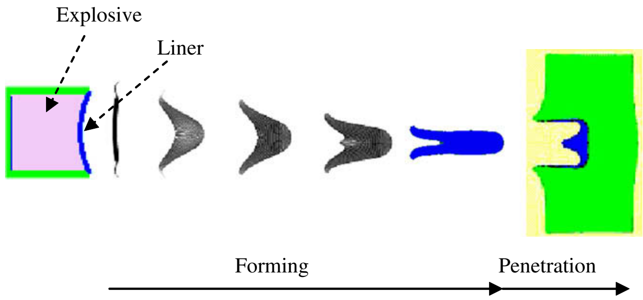

Fig. 1. Schematic illustration of the formation process of an explosive forming projectile (EFP) and the final morphology of the penetration into the target.

models, Johnson–Cook (JC), Zerilli–Armstrong (ZA), Voyiadjis and Abed (VA), and Preston–Tonks–Wallace (PTW), for the case of the plastic deformation of tantalum. Through the comparison of each model prediction with existing experimental data elsewhere, it will be shown that while none of above models is appropriate in wide ranges of strain, strain rate, and temperature, the PTW model has good fitting capability for high-strain rate and high temperature, provided the strain-hardening term is modified properly. The modification process as well as the capability of the modified PTW for tantalum in wide ranges of strain, strain rate, and temperature, will be presented.

2. Review of four constitutive models

The effects of strain, strain rate, and temperature on the flow stress are not independent of each other: there exists an interaction effect. Thus, accurate determination of the flow stress as a function of the above-mentioned three parameters is not an easy task. Also, the minimum number of fitting coefficients is preferred, because in most cases the coefficients are determined from experiment. The four constitutive models compared in the current work are reviewed hereinafter.

2.1. JC model [3]

The JC model describes the plastic flow stress by the relation,

$$\overline{\sigma} = (A + B\overline{\varepsilon}^n)\left(1 + C\ln\left(\frac{\dot{\overline{\varepsilon}}}{\dot{\overline{\varepsilon}}_0}\right)\right)\left(1 - T^{*m}\right) \tag{1}$$

where, $\overline{\varepsilon}$ is the equivalent plastic strain, $\dot{\overline{\varepsilon}}$ is the equivalent plastic strain rate, $\dot{\overline{\varepsilon}}_0$is the reference strain rate which is usually set to 1 s^{-1}, and T^* is the homologous temperature defined as $(T - T_{\mathrm{room}})/(T_{\mathrm{melt}} - T_{\mathrm{room}})$. T is the absolute temperature and A, B, n, C, and m are material constants. In this model, strain hardening, strain-rate hardening, and thermal softening are taken into account as a multiplication form. The expression in the first set of parenthesis gives the flow stress as a function of equivalent plastic strain for unit strain rate and $T^* = 0$. The expressions in the second and third sets of parentheses represent the effects of the equivalent plastic strain rate and temperature, respectively. The JC model has some shortcomings. First, the linear relation of the flow stress is predicted with temperature while such behavior is often not the case in practice, especially at high temperature. The flow stress at high temperature does not decrease linearly as the temperature increases. Second, the model predicts a linear increase of flow stress with log strain rate. However, some materials, such as tantalum, are known to show an abrupt increase in flow stress at a certain strain rate [5]. Finally, Eq. (1) gives a negative flow stress when the plastic strain rate is nearly zero.

2.2. ZA model for BCC materials [2]

The ZA model has different forms of constitutive relation for body-centered-cubic (BCC) materials and face-centered-cubic (FCC) materials. The constitutive equation for BCC materials such as tantalum is as follows:

$$\overline{\sigma} = C_0 + C_1\exp(-C_3T + C_4T\ln\dot{\overline{\varepsilon}}) + C_5\overline{\varepsilon}^n \tag{2}$$

where, C_0, C_1, C_3, C_4, C_5, and n are material constants. The first term C_0 is related to Hall–Petch relation $\sigma_0 + \mathrm{kd}^{-1/2}$, where d is the grain size of the material. In this model, it is presumed that the work hardening is independent of temperature and strain rate. This means that the thermally activated movement of a dislocation is independent of the plastic strain. The saturation of strain-rate hardening, however, is related to the temperature softening. The power-law stress-strain relationship in Eq. (2) exhibits a continual work hardening without saturation of flow stress at a large strain.

2.3. VA model [5]

To improve the prediction capability of flow stress at high-strain rates and temperatures Voyiadjis and Abed [5] modified the ZA model as follows:

$$\overline{\sigma} = \widehat{Y}\left[1 - \left(\beta_1T - \beta_2T\ln\dot{\varepsilon}_P\right)^{1/q}\right]^{1/p} + B\varepsilon_P^n + Y_a \tag{3}$$

where, $\widehat{Y}$, β_1, β_2, Y_a, B, p, q, and n are material constants. The last two terms are athermal components of flow stress and they are the same forms as the ZA model in Eq. (2). The first term is thermal flow stress and it is related to the strain rate and temperature. The first term was modified from the ZA model and derived by using the concept of thermal activation energy as well as the dislocation interaction mechanism. The mobile dislocation density evolution was also taken into account. By modifying the thermal component of flow stress, the prediction capability at high-strain rates and temperatures is reportedly improved [5].

2.4. PTW model [6]

Thermal activation mechanism of dislocation has a significant influence on the deformation by weak shocks of strain rate up to 10^5 s^{-1}. The strain rate in explosively driven deformations or in high-velocity impacts is sometimes much higher than 10^5 s^{-1}, and thus the plastic constitutive model based on only the thermal activation mechanism can result in a significant error. In order to model the material behavior accurately at a strain rate up to 10^{12} s^{-1}, Preston et al. [6] proposed a plastic constitutive model considering nonlinear dislocation drag effects that are predominant in a strong shock regime. The model is given by

$$\begin{aligned}\widehat{\tau} = \widehat{\tau}_s + \frac{1}{p}\left(s_0 - \widehat{\tau}_y\right)\ln\Bigg[1 - \left[1 - \exp\left(-p\frac{\widehat{\tau}_s - \widehat{\tau}_y}{s_0 - \widehat{\tau}_y}\right)\right] \\ \times \exp\left\{-\frac{p\theta\overline{\varepsilon}}{\left(s_0 - \widehat{\tau}_y\right)\left[\exp\left(p\frac{\widehat{\tau}_s - \widehat{\tau}_y}{s_0 - \widehat{\tau}_y}\right) - 1\right]}\right\}\Bigg]\end{aligned} \tag{4}$$

where, $\widehat{\tau}$ is a normalized flow stress ($=\tau/G$ where τ is the shear stress and G is the shear modulus), and $\widehat{\tau}_S$ and $\widehat{\tau}_Y$ are the normalized work hardening saturation stress and normalized yield stress, respectively. The variables, p, θ, and s_0 are dimensionless material constants. $\widehat{\tau}_S$ and $\widehat{\tau}_Y$ are defined as,