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Analyses of impact motions of harmonically excited systems having rigid amplitude constraints

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Abstract

Two harmonically excited systems having symmetrical rigid constraints are considered. Repeated impacts usually occur in these systems due to the rigid amplitude constraints. The impact forms occurring in two systems are different, i.e., the components of one system collide with each other, and one of components of the other system collides with rigid obstacles. Dynamics of these systems are studied with special attention to Neimark–Sacker bifurcations associated with several periodic-impact motions. Period-one double-impact symmetrical motions and associated Poincaré maps of two systems are derived analytically. Stability and local bifurcations of the period-one double-impact symmetrical motions are analyzed by using the Poincaré maps. Neimark–Sacker bifurcations associated with several periodic-impact motions are found by numerical simulation, and the corresponding routes from quasi-periodic impact motions to chaos are also stated. The influence of the clearance and excitation frequency on symmetrical double-impact periodic motion and bifurcations is analyzed. Studies show that the vibratory systems having symmetrical rigid amplitude constraints may exhibit complex and rich quasi-periodic impact behavior under different system parameter conditions.

Keywords: Vibration; Impact; Periodic motion; Quasi-periodic motion; Neimark–Sacker bifurcation

1. Introduction

Impact oscillators arise whenever the components of a vibrating system collide with rigid obstacles or with each other. Such systems with impacts exist in a wide variety of engineering applications, particularly in mechanisms and machines with clearances, gaps or stops. The principle of operation of vibration hammers, impact dampers, inertial shakers, pile drivers, offshore structures, milling and forming machines etc., is based on the impact action for moving bodies. With other equipment, e.g., machines with clearances, heat exchangers, gears, rolling railway wheelset, piping systems and so on, impacts also occur, but they are undesirable as they bring about failures, strain, shorter service life and increased noise levels. The research into vibro-impact dynamics has important significance in noise suppression, reliability analysis and optimum design of machines with clearances or rigid obstacles. The trajectories of such systems in phase space have

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discontinuities caused by the impacts. The presence of the non-linearity and discontinuity complicates the dynamic analysis of such systems considerably, but it can be described theoretically and numerically by discontinuities in good agreement with reality. Compared with single impact, the vibro-impact dynamics is more complicated, and hence, has received great attention. The large interest in analyzing and understanding the performance of such systems is reflected by vast and ever increasing amount of research effort devoted in this area. Some important problems on vibro-impact dynamics, such as stability and global bifurcations [1–8], visco-elastic impact vibration [9], grazing singularities of impact mapping [11–17], chattering impacts [18], rattling impacts [19], quasi-periodic impacts [20–25], controlling chaos [26,27] and experimental study [28,29], etc., have been studied in the past several years. Along with the basic research into vibro-impact dynamics, the research into application to these systems are developed, e.g., wheel–rail impacts of railway coaches [29–31], impact noise analysis [32,33], inertial shakers [34], vibration hammer [35], Jeffcott rotor with bearing clearance [36,37], ground moling dynamics [38], impact damper [39–41] and gears [42–44], etc. Two harmonically excited systems having symmetrical rigid constraints are considered in the paper. The impact forms occurring in the two systems are different, i.e., the components of one system collide with each other, and one of components of the other system collides with rigid obstacles. Dynamics of these systems are studied with special attention to Neimark–Sacker bifurcations associated with several periodic impact motions. Period-one double-impact symmetrical motions and associated Poincaré maps of two systems are derived analytically. Stability and local bifurcations of period-one double-impact symmetrical motions are analyzed by using the Poincaré maps. Neimark–Sacker bifurcations associated with several periodic-impact motions are found by numerical simulation, and the corresponding routes from quasi-periodic impact motions to chaos are also stated. The influence of the clearance and excitation frequency on symmetrical double-impact periodic motion and bifurcations is analyzed. The studies show that the vibratory systems having symmetrical rigid amplitude constraints may exhibit complex and rich quasi-periodic impact behavior under different system parameter conditions.

2. Mechanical models

The mechanical models for two harmonically excited systems having symmetrical rigid constraints are shown schematically in Figs. 1(a) and (b), respectively. We first analyze the mechanical model of an impact damper shown in Fig. 1(a). Ideally, this model comprises two systems: the primary one consists of a rigid body with mass M_1, a linear spring with stiffness K, and a viscous damper with damping constant C, while the secondary system is a point mass M_2. The clearance between the two masses is denoted by $2D$, and it is assumed that no friction exists between these two masses. In addition, an external harmonic force, $P_1\sin(\Omega T+\tau)$, is applied on the primary mass. Displacements of the masses M_1 and M_2 are represented by X_1 and X_2, respectively. The point mass M_2 impacts mutually with the primary mass M_1 when $|X_2(T)-X_1(T)| = D$. The impact is described by the conservation law of momentum and a coefficient of restitution r, and it is assumed that the duration of impact is negligible compared to the period of the force.

The motion processes of the system, between consecutive impacts occurring at the baffle A_1, are considered. Between any two consecutive impacts, the time T is always set to zero directly at the starting point (the baffle A_1), and the phase angle τ is used only to make a suitable choice for the origin of time in the calculation. The

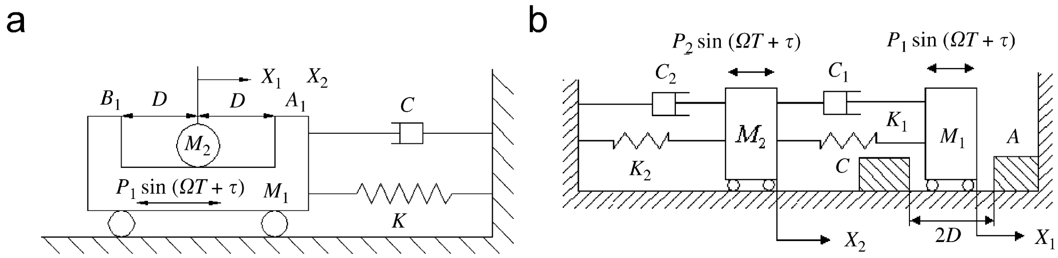

Fig. 1. Schematics of two harmonically excited systems having symmetrical rigid constraints: (a) the impact damper; and (b) the vibratory system with a clearance.