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Near-simultaneous impacts

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Abstract

This work considers a three-mass collision model with compliant contacts. If the masses are initially separated by a sufficient distance, the collision sequence is pairwise, that is, comprised of a series of pairwise impacts. The final velocities of each mass are then independent of the specific initial spacing. In contrast, if the initial spacing between the masses is small the three masses interact simultaneously and the final velocity state depends not only on the initial velocities, but also on the initial configuration of the system. Given the impact duration and coefficient of restitution for a pairwise collision, which can be determined from appropriate differential collision models, a two-dimensional map is derived to describe those initial conditions that lead to pairwise sequences in the three mass system. Finally, for a specific nonlinear compliance model the variation in the final velocities are characterized in terms of the initial configuration of the system.

Keywords: Multiple impact; Sequential collisions; Contact compliance

1. Introduction

Multibody collisions are widely regarded as much more complex than the more common instances of pairwise impacts. However, multibody collisions play an important role in the dynamics of granular systems and the transmission of impact forces through interconnected mechanical systems. Given the pre-collision state of the system, the post-collision state can be

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found from detailed continuum mechanics (CM) models with appropriate constitutive assumptions, but only with significant complexity and/or computational cost. Common approaches for the development of simplified multibody impact problems include force–displacement (FD) models, with time as the independent variable, and impulse–momentum (IM) approaches, which use the component of impulse normal to the contact plane as the independent quantity. Loosely speaking, IM models can be thought of as reduced FD models, which are in turn reduced from CM models.

IM models can be applied without regard for the "real" time duration of the collision because the normal impulse is used as the independent variable. Hence, these models are commonly applied to rigid-body simulations—the noncollisional behavior of each object is assumed to be described by the laws of rigid-body dynamics and a collision is characterized by an instantaneous finite jump in the velocities of the system. In this context, uniqueness of a collision model implies that the post-collision velocities are uniquely determined by their pre-collision values. For unconstrained pairwise collisions, only one collision impulse exists and several unique IM models have been developed, including kinematic (Newtonian) and kinetic (Poisson) restitution. However, for multibody collisions, IM models also require assumptions on how and when the different reaction impulses are applied to neighboring bodies. Without these additional assumptions, the IM models developed for pairwise collisions cannot uniquely determine the post-collision velocities. Moreover, different secondary assumptions can lead to post-collision velocities that do not match experimentally observed results [1,2].

Within IM models, the final velocities in a multi-ball chain depend on the assumed timing, in both duration and initiation, of the reaction impulse arising from each pair of colliding particles. One common limit of these timings, defined as a sequential impact, occurs in a multi-ball chain as a sequence of pairwise interactions, each isolated from the remainder of the system—one pair of particles collides, followed by a second pair, etc. This specific timing of the impulses has been used by many researchers to predict the outcome of multi-ball chains and for many systems has been shown to agree with experimental results [2–4]. In contrast, a second limiting case, defined as simultaneous impact, occurs when the time of maximum compression is identical for each pairwise interaction [5]. In terms of an appropriate IM model, every individual pairwise collision model is applied simultaneously.

In general, multiple impacts are neither sequential nor simultaneous and depends on both the material parameters as well as the initial state of the system. In FD models the evolution of the contact forces in time is determined as part of the solution process. In a three-mass chain, Newby showed that certain final velocities are a nonunique function of the contact parameters [6] for an FD model based on linear elastic compliances. To uniquely specify the contact parameters from the final velocities (defined as the *inverse scattering problem*), he proposed to use time correlations between the incident and scattered particles. Ivanov [2] considered the multiple impact problem as a stochastic process and represented the solution as a random variable. Stronge also considered the role of spatial gradients in the contact compliance ("wave speed") on the post-impact velocities within an FD model [5,7]. In particular, for compliance that increases away from the initial impact location, the collision becomes sequential while if the compliance decreases, the collision becomes simultaneous. The contact compliance is also related to the timescale of an individual collision event. Increasing compliance implies a lengthening duration for the impact. Ceanga and Hurmuzlu [8] have developed an IM model to calculate the post-impact velocities,