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Shear band spacing in thermoviscoplastic materials

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Abstract

A closed-form expression for shear band spacing in strain-hardening, strain-rate-hardening and thermally softening thermoviscoplastic materials is derived by studying the stability of a homogeneous solution of equations governing its simple shearing deformations. The wavelength of the perturbation that maximizes its initial growth rate is assumed to determine the shear band spacing, L_s. The dependence of L_s upon various material parameters and the nominal strain rate, $\dot{\varepsilon}$, is delineated. When written as $L_s = A_1 k^{\chi_1}$ or $A_2 \dot{\varepsilon}^{\chi_2}$ where A_1 and A_2 are parameters and k is the thermal conductivity, it is found that $\chi_2 \simeq -0.787$ and χ_1 depends upon the strain-rate hardening exponent m; $\chi_1 \simeq 0.5$ for $m \simeq 10^{-6}$ and $n \simeq 0.011$, decreases rapidly to 0.21 for $m \simeq 10^{-4}$ and $n \simeq 0.011$, and then increases slowly to 0.25 for $m \simeq 0.05$ and $n \simeq 0.011$. However, for $m = 0$ and $n \neq 0$, $\chi_1 = 1$.

Keywords: Simple shearing; Perturbation method; Analytical expression

1. Introduction

The study of adiabatic shear bands (ASBs) is important since they precede ductile failure in most materials deformed at high strain rates. An ASB is a narrow region, usually a few micrometers wide, with plastic strains often exceeding 1. Even though Tresca [1] observed these in 1880, research in this area intensified with the work of Zener and Hollomon [2] who not only

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observed ASBs during the punching of a hole in a low carbon steel plate but also postulated that they initiate when a material point becomes unstable due to softening caused by heating overcoming its hardening induced by strain and strain-rate effects.

The perturbation technique to analyze the stability of transient simple shearing deformations of a thermoviscoplastic body has been employed by Bai [3]. A homogeneous solution of the problem at time t_0 is perturbed by an infinitesimal amount and governing equations are linearized with coefficients evaluated at time t_0. Bai [3] derived conditions necessary for the homogeneous solution to become unstable and also computed the wavelength of the perturbation that maximized its initial growth rate; he defined a characteristic length in terms of this wavelength. This and other works are summarized in books by Bai and Dodd [4] and Wright [5], in the review article of Tomita [6], and in books or special issues of journals edited by Zbib et al. [7], Armstrong et al. [8], Perzyna [9], Batra and Zbib [10] and Batra et al. [11]. Batra and Chen [12] have pointed out that for locally adiabatic simple shearing deformations of a thermoviscoplastic body, the critical strain evaluated from Bai's criterion equals that given by the Considerѐ condition [13] which states that a structure becomes unstable when the load required to deform it is maximum. For the simple shearing problem, the Considerѐ criterion is equivalent to the shear stress attaining its maximum value. Batra and Kim [14] have shown through numerical experiments that the thermal conductivity has a negligible influence on the time of initiation of an ASB but affects the post-localization process. Thus, even when heat conduction is considered, the critical strain computed from Bai's instability criterion agrees with that from the Considerѐ condition. Wei and Batra [15] have shown that this holds even when damage evolution is also considered mainly because not much damage has evolved till the time of initiation of an ASB.

Wright and Ockendon [16] also examined the growth of infinitesimal perturbations superimposed upon a homogeneous solution and ignored strain-hardening effects. They postulated that, in an infinite body, perturbations growing at different sites will not merge and result in multiple ASBs. They equated the shear band spacing to the wavelength of the instability mode that maximizes the initial growth rate of infinitesimal perturbations. The shear band spacing so determined can be related to Bai's [3] characteristic length. For materials obeying the constitutive relation $\sigma = \sigma_0(1 - \alpha(\theta - \theta_0))(\dot{\gamma}/\dot{\gamma}_0)^m$, Wright and Ockendon's expression for the shear band spacing is $L_{\mathrm{WO}} = 2\pi(m^3kc/(\dot{\gamma}^3\alpha^2\sigma_0))^{1/4}$. Here σ is the shear stress, γ the shear strain within the ASB, α the thermal softening coefficient, m the strain-rate-hardening exponent, θ the present temperature within the ASB, c the specific heat, k the thermal conductivity and $\dot{\gamma}_0$ the nominal strain rate. Values of $\dot{\gamma}$ and θ in an ASB need to be estimated in order to compute L_{WO}. Molinari [17] considered strain hardening effects and defined the shear band spacing as $L_{\mathrm{M}} = \inf_{t_0 \geqslant 0} 2\pi/\xi_{\mathrm{m}}(t_0)$, where ξ_{m} is the wavelength of the perturbation introduced at time t_0 that has the maximum growth rate at t_0. For materials obeying the constitutive relation $\dot{\gamma} = \mu_0^{-1/m}\sigma^{1/m}(\gamma + \gamma_i)^{-n/m}\theta^{-v/m}$, Molinari's approximate expression for the shear band spacing is

$$
\begin{aligned}
L_{\mathrm{M}} &= L_0(1 + (3\rho c\partial\dot{\gamma}/\partial\gamma)(4\beta\sigma^0\partial\dot{\gamma}/\partial\theta))^{-1} \\
&= 2\pi((m^3kc(\theta^0)^2/(\beta^2\dot{\gamma}^3v^2\sigma^0(1+m)))^{1/4}\left(1 + \frac{3\rho cn}{4\beta v\sigma^0}\frac{\theta^0}{\gamma+\gamma_i}\right)^{-1},
\end{aligned}
$$

where L_0 is the shear band spacing for $n = 0$, σ^0 the shear stress and θ^0 the temperature at time t_0 in the homogeneous solution, β the fraction of plastic working converted into heating, and ρ the