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Development of an LEFM dynamic crack criterion for correlated size and rate effects in concrete beams

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A B S T R A C T

The strength of concrete under severe dynamic loading depends on the specimen size and the loading rate. Although the size effect, under quasi-static loading, has been explained by the size-dependent strain energy rate, the main causes of the size and rate effects for dynamic loading cases have not been clarified. In this study, a linear elastic fracture mechanics (LEFM) dynamic crack criterion for a notched three-point bend specimen is developed to explain the size and rate effects, and the possible correlation of these effects. This was achieved by using energy balance, force equilibrium, and Griffith's crack model. From the proposed LEFM dynamic crack criterion, it was shown that (1) the kinetic energy rate seems to be the main cause of the rate effect, (2) the size and rate effects are not independent phenomena.

1. Introduction

It is well known that, under quasi-static loadings, the strain energy rate in concrete specimens increases with the increase of the structural size, which induces a size effect [1–3]. However, this quasi-static form of the size-dependent strain energy rate cannot be adopted for describing the size and rate effects under dynamic loading, where kinetic energy should be considered. Since it is difficult to calculate strain and kinetic energies in dynamic fracture mechanics due to the presence of discontinuities, such as cracks and flaws, empirical equations that express the strength enhancement by loading or strain rate were used to represent the rate effect [4–9]. However, since an empirical approach cannot provide an objective solution, an effort should be made to establish energy based dynamic crack criterion by employing fracture mechanics.

Shah and John [10] conducted Charpy impact tests on single edge notched beams with a resistive type of thin foil gage called KRAK gage, which can measure the propagation of a single continuous surface crack, and reported that the amount of pre-peak crack growth decreases with the increase in loading rate. The decrease in pre-peak crack growth at high strain rates indicated that the fracture process zone decreased, as the loading rate increased. Similar observations were made by other investigators [11,12] based on the measurements of load–deflection curves of beams, and of stress–strain curves in compressive specimens, respectively. The negligible pre-peak crack growth at high loading rate indicates that the crack starts to propagate at the peak load. Hence, LEFM could be valid for studying size and rate effects of concrete in high loading rate cases.

In LEFM, the rate effect can be explained by the delay of crack propagation, which may be caused by the kinetic energy term in a dynamic crack criterion. If the kinetic energy rate term is preceded by a negative sign, additional strain energy rate is required to reach the critical energy rate in the dynamic crack criterion. This additional strain energy rate may explain the rate effect in concrete under high loading rates.

In the present study, an LEFM dynamic crack criterion for a notched three-point bend specimen was developed by hypothesizing that the rate effect may originate from the kinetic energy term with a negative sign in a dynamic crack criterion. The proposed LEFM dynamic crack criterion will provide the physical explanation of how the loading rate and the structural geometry are coupled to form both the size and rate effects, and give reliable structural analysis results at high loading rates.

2. Review of the single degree of freedom model

Closed form solutions of continuous structural behavior under dynamic loading should be derived to develop a dynamic crack criterion. However, it is difficult to formulate closed form solutions due to the presence of cracks. Thus, a simplification of the mathematical governing equations of motion is inevitable. One simplified

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Notation

a	crack or notch length
B	beam width
DF	dynamic factor
DIF	dynamic increase factor
E	modulus of elasticity
I	area moment of inertia
L	span length
P	applied force
P_c	critical bending load
u	deflection in an SDOF model
U_e	work
U_k	kinetic energy
U_s	strain energy
W	beam depth
Y	first mode shape
φ	first mode slope
ε	strain at the bottom of the mid-span
ρ	mass density
λ	a/W
Ψ	shape function
ν	Poisson's ratio
γ_s	surface energy

method is the single degree of freedom approach (SDOF), in which the mass and stiffness are formulated to have the same energy distribution as in the continuous closed form solution. It may give a general idea of the relationships for the strain and kinetic energy rates, and their possible contributions to the size and rate effects, although the SDOF model can express only a first mode of dynamic structural behavior.

Kishimoto's closed form solution [13] was adopted for the first mode. This approach was derived for the normal modes of deflection and bending slope of a notched Timoshenko beam (shown in Fig. 1) subjected to an arbitrary dynamic concentrated load by assuming that the crack affects only the area moment of inertia of the cracked section. In Fig. 1, P is the applied force, B is the beam width, W is the beam depth, L is the span length, and a is the crack or notch length.

By assuming that the first mode may govern the dynamic behavior of a single notched beam, the continuous Timoshenko beam problem can be reduced to an SDOF model with the following equivalent mass and stiffness for plane strain condition:

$$M_e = \frac{2\rho BWL}{Y^2(1/2)}\int_0^{1/2}\left[Y^2(\xi)+\frac{I}{A}\varphi^2(\xi)\right]d\xi \tag{1}$$

$$K_e = \frac{48EI}{L^3}\left(1+\frac{6}{L/W}\Psi+12q^2\right) \tag{2}$$

where Y and φ are the first mode deflected shape and slope, respectively, in Kishimoto's solution [13], ρ is the mass density, I is the area moment of inertia, and E is the modulus of elasticity. Also,

$$q = \frac{1}{L}\sqrt{\frac{(12+11\nu)I}{5BW}} \tag{3}$$

and Ψ is the following shape function [14]:

If $L/W = 4$:

$$\Psi = \left(\frac{\lambda}{1-\lambda}\right)\left(5.58-19.57\lambda+36.84\lambda^2-34.94\lambda^3+12.77\lambda^4\right) \tag{4}$$

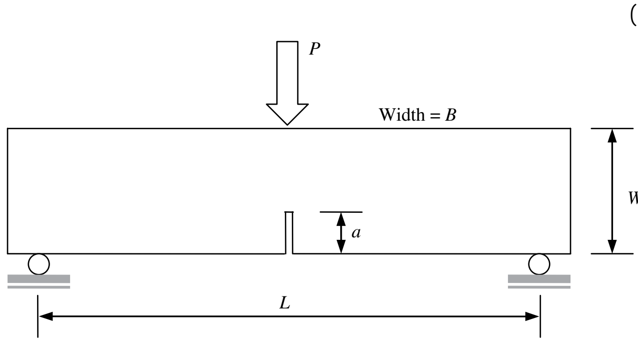

Fig. 1. Three-point bending of a single notched beam.

If $L/W = 8$:

$$\Psi = \left(\frac{\lambda}{1-\lambda}\right)\left(5.755-19.63\lambda+36.98\lambda^2-35.39\lambda^3+12.945\lambda^4\right) \tag{5}$$

where ν is Poisson's ratio, and λ is the ratio given by a/W.

It should be noted that the equivalent mass and stiffness are functions of the crack length a, because the mode shapes (Y and φ) and Ψ depend on the crack length.

3. Development of the LEFM dynamic crack criterion

An LEFM dynamic crack criterion for an SDOF system can be derived by applying the energy balance and force equilibrium equations. For the energy balance, the work rate should be equal to the sum of the rates of strain, kinetic and surface energies, as shown in Eq. (6), in which the surface energy is the required energy to propagated unit crack area based on Griffith's crack model.

$$\dot{U}_e = \dot{U}_s + \dot{U}_k + 2\gamma_s B\dot{a} \tag{6}$$

in which upper dots represent time derivatives, and γ_s is the surface energy. U_e, U_s, and U_k are the work, strain, and kinetic energies, respectively. For an SDOF model, rates of these energies can be described by the following expressions:

$$\dot{U}_e = P\dot{u} \tag{7}$$

$$\dot{U}_s = \frac{d}{dt}\left(\frac{1}{2}K_e u^2\right) = K_e u\dot{u}+\frac{1}{2}\frac{dK_e}{da}u^2\dot{a} \tag{8}$$

$$\dot{U}_k = \frac{d}{dt}\left(\frac{1}{2}M_e\dot{u}^2\right) = M_e\ddot{u}\dot{u}+\frac{1}{2}\frac{dM_e}{da}\dot{u}^2\dot{a} \tag{9}$$

in which u is the deflection in an SDOF model, and P is the applied force.

In addition to the energy balance equation, the following force equilibrium also must be satisfied:

$$\frac{d}{dt}(M_e\dot{u})+K_e u = P \tag{10}$$

which yields the following expression:

$$M_e\ddot{u}+\frac{dM_e}{da}\dot{a}\dot{u}+K_e u = P \tag{11}$$