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Elastic stress transmission in cellular systems—Analysis of wave propagation

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Abstract

A closely packed array of thin-walled rings constitutes an idealisation of a cellular structure. Elastic waves propagating through such structures must do so via the ring (cell) walls. A theoretical investigation into the propagation of elastic stresses in thin-walled circular rings is undertaken to examine the nature of wave transmission. Three modes of motion, corresponding to shear, extensional and flexural waves, are established and their respective velocities defined by a cubic characteristic equation. The results show that all three waves are dispersive. By neglecting extension of the centroidal axis and rotary inertia, explicit approximate solutions can be obtained for flexural waves. Employment of Love's approach for extensional waves [Love AEH. A treatise on the mathematical theory of elasticity, 4th ed. New York: Dover Publications; 1944. p. 452–3] enables approximate solutions for shear waves to be derived. The three resulting approximate solutions exhibit good agreement with the exact solutions of the characteristic equation over a wide range of wavelengths. The effects of material property, ring wall thickness and ring diameter on the three wave modes are discussed, and the results point to flexural waves as the dominant means of elastic energy transmission in such cellular structures. Wave velocities corresponding to different frequency components determined from experimental results are compared with theoretical predictions of group velocity for flexural waves and good correlation between experimental data and theory affirms this conclusion.

Keywords: Elastic wave propagation; Cellular structures; Flexural waves; Dispersion

1. Introduction

Tightly packed arrays of circular rings can be considered idealised representations of cellular structures and facilitate investigation of elastic wave propagation in cellular systems. A previous experimental study has been conducted to examine the nature of elastic stress transmission through arrays of metal rings [2]. To obtain further insights into the characteristics of such wave propagation, a theoretical analysis of elastic wave motion in thin-walled circular rings is undertaken and the results are compared with the experimental data obtained previously.

Elastic wave propagation in a curved waveguide has been studied by a number of researchers; Love [1,3] developed an approximate theory for elastic wave motion in a helical wire and a ring. He neglected rotary inertia and radial shear, and uncoupled flexural from extensional motion—the centroidal axis of the ring was assumed to be inextensible when considering the flexural motion, while flexural behaviour was ignored when considering extensional motion. Waltking [4] extended Love's theory [1,3] by considering extension of the centroidal axis, while Philipson [5] incorporated rotary inertia. These approximate theories were only applicable to low frequencies (long wavelengths); for high frequencies, the effect of

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radial shear must be considered. Morley [6] and Graff [7] incorporated all three factors—centroidal axis extension, rotary inertia and radial shear—in their investigation of wave motion in a curved beam, while Wittrick [8] studied wave propagation in a helical spring of small curvature. They all predicted three modes of motion—radial shear, flexure and axial extension. Theories by Morley [6] and Graff [7] are applicable to thick rings, while the theory of Wittrick [8] is only valid for thin-walled rings. Haines et al. [9] provided exact solutions for wave propagation in thin-walled rings under conditions of generalised plane stress and discussed the characteristics of shear, extensional and flexural waves. He also developed an approximate theory [10] for thin-walled rings, whereby the governing equation is equivalent to Wittrick's analysis [8] for a helical coil with a 0° pitch angle. From comparison with exact theory, Haines showed that with an appropriate choice of a shear correction factor, the approximate theory is accurate for the same frequency range that Timoshenko beam theory applies for straight bars.

For specific initial and boundary conditions, the differential equations governing wave motion in curved waveguides have also been solved numerically. Phillips, Crowley and Taylor [11,12] used the method of characteristics to solve Morley's equation for motion of a curved beam subjected to pulse loading, and the numerical results displayed good agreement with photo-elasticity experiments. Shim and Quah [13,14] employed Timoshenko-type theory, coupled with the method of characteristics, to analyse the elastic response of a circular ring subjected to radial impact, and examined the effects of ring curvature, cross-section geometry and impact duration on the resulting shear force, axial force and bending moment distributions, as well as ring deformation. Their theoretical results exhibited good correlation with experiments involving impact on brass and aluminium rings.

A circular ring is the basic unit of the ring arrays studied in previous experiments [2], and propagation of elastic disturbances through such arrays can be examined via wave analysis. In this investigation, Timoshenko-type theory is applied to analyse wave propagation in a thin-walled circular ring, with the perspective that this will be useful in understanding the transmission of elastic loading through cellular structures of low relative density. An objective is to determine which mode of motion—shear, extension or flexure—conveys most of the elastic energy. The governing equation for wave motion is first derived and a cubic characteristic equation is established using harmonic wave analysis. Standard iterative procedures (Newton–Raphson method) are applied to obtain the velocities corresponding to the three modes of wave propagation; wave dispersion characteristics are then examined. Approximate solutions in explicit form for wave velocities are also developed, and the effects of material property, ring wall thickness and ring diameter on the phase and group velocities are studied. Finally, experimental results for wave speeds corresponding to different frequencies are compared with theoretical predictions for flexural wave group velocities.

2. Theoretical analysis for elastic wave propagation in a thin circular ring

2.1. Governing equations and propagation of harmonic waves

To investigate elastic wave propagation in a circular ring, a simplified theoretical analysis is formulated via extension of the well-known Timoshenko beam theory. The basic assumption is that plane cross-sections remain plane after deformation, but are not necessarily perpendicular to the centroidal axis. This implies that the effect of shear deformation is considered and that the total transverse displacement of the beam axis is obtained by summing displacements due to bending and shear. The effect of rotary inertia is also included.

Only rings with a rectangular cross-section are analysed in this investigation, to facilitate comparison with results from previous experiments [2]. Fig. 1 illustrates the displacements and forces acting on the cross-section of a ring, and only in-plane motion is considered. The circumferential and radial displacements of the centroid of the ring cross-section are denoted respectively by u and v, while rotation of the cross-section is defined by Θ. The corresponding axial (circumferential) force on the cross-section is P, the radial shear force is Q and the bending moment is M. Consider the motion of an element in the ring; application of elastic stress–strain relationships yields the following set of six equations linking the forces to the displacements:

$$\begin{aligned}
&\frac{\partial P}{\partial s}-\frac{Q}{R}=m\frac{\partial^2 u}{\partial t^2},\\
&\frac{\partial Q}{\partial s}+\frac{P}{R}=m\frac{\partial^2 v}{\partial t^2},\\
&\frac{\partial M}{\partial s}+Q=mk^2\frac{\partial^2 \Theta}{\partial t^2},
\end{aligned} \tag{1}$$