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Design analysis of sandwiched structures experiencing differential thermal expansion and differential free-edge stretching

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ABSTRACT

Compared to the theory of elasticity solutions the strength of material solutions offer closed form solutions that is favoured by practising engineers for performing design analysis. However, the existing strength of material solutions for a sandwich structure experiencing differential thermal strains have principally ignored the free edge conditions; and for the very limited publications that have enforced the free edge conditions, the solutions have been inaccurate. Understandably, design analysis using such solutions is unreliable. This manuscript describe a solution technique that enforces the nil shear stress condition at the free edge using a high power exponential function resulting in a simple yet accurate closed form solutions for the interfacial shear stress. The interfacial peeling stress is made up of two components: the mean and the amplitude of variations of the transverse normal stress in the bonding layer; the latter is the dominant component whose magnitude is linearly proportional to the gradient of the interfacial shear stress. Following validation by finite element analysis, design analysis for debonding, fracturing, and out-of-plane deformation are performed using the concise closed form solutions.

1. Introduction

Many engineering structures and assemblies, ranging from composites structures used in aircrafts to microelectronic/optoelectronic assemblies are made of sandwiched construction. A sandwiched structure is made of two adherends that are bonded with a bonding layer. Thermal stresses in a multi-layer structure has been a subject of great interest. The earliest analysis could be traced to Stoney [1] who analysed the in-plane stress on a film deposited on a substrate. Timoshenko [2] was interested in the out-of-plane deformation of two dissimilar metals used as thermostat. The analysis of interfacial thermal stresses in multilayer structures is far more challenging and has been a subject of intense study. Aleck [3] is believed to have been the first to investigate the interfacial thermal stresses in a bi-material structure – a film bonded to a rigid substrate. Since then, the analyses have taken two separate paths: the theory-of-elasticity approach, and the strength-of-material approach. The former describes the stress field using stress functions, which for a two-dimensional body are most frequently the Airy stress functions. This returns rigorous and well-substantiated solutions including describing the singular stress field at the free edge of interfaces [4–6]; however, the solutions are usually too complicated to be presented in the form of closed-form solution and almost inadvertently requires the use of numerical techniques for its solutions [7–21]. The strength-of-material approach breaks down the constituting bodies into structural elements that are prescribed with assumed displacement behaviour. The assumptions have simplified significantly the analysis and could lead to closed-form solutions [22–39]. Among these works, Taylor and Yuan [22] is believed to be the first to present the interfacial shear stress in the closed-form solution of exponential or hyperbolic function while Suhir [26] is credited for pioneering the compliance technique.

The problem of thermal stress has been particularly troubling the microelectronics and optoelectronics assembly communities because of the rapid pace at which new designs of these assemblies and new materials are introduced. Compared to the aerospace industry, the microelectronics and optoelectronics industry also enjoy much more design flexibility. As a consequence, design engineers are expected to explore vast design space within a relative short development process. Comparing to numerical analysis such as the finite element analysis, the complex solutions from theory-of-elasticity offers little advantages. On the other hand, the simple closed formed solutions from strength-of-material offers valuable physical insights and is particularly attractive for design explorations. It is no coincidence that the bulk of the strength-of-material solutions listed above have originated from the microelectronic/optoelectronic communities.

However, a major setback for the strength-of-material solutions is their inability to model accurately the magnitude of the stresses

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Nomenclatures

Subscript $\#_i$	Subscript #1 & #2 are outer members and #3 is a bonding layer.
D_i, E_i, G_i, h_i	Flexural rigidity, elastic modulus, shear modulus, thickness of member #*i*.
$\overline{D}_i, \overline{D}_e$	Flexural compliance of member #*i*, effective flexural compliance of the sandwich structure.
f_p, f_a, f_b	Interfacial peeling stress, mean, amplitude of the interfacial peeling stress.
f_s	Interfacial shear stress.
F_i	*x*-directional sectional traction acting along the centroid axis of member #*i*.
L	Half length of the sandwich structure.
Q_a, Q_b	Sectional shear forces corresponding to f_a and f_b, respectively.
u_i, w_i	*x*-directional, *z*-directional displacement of the centroid axis of member #*i*.
α_z, β	Characteristic constants for through-thickness displacement, shear deformation of the sandwich structure.
α_i; α_{ij}	Coefficient of thermal expansion; $\alpha_{ji} = \alpha_j - \alpha_i$.
κ_{si}, κ_s	Shear compliances of member #*i*, of the sandwich structure between the centroid axes of members #1 and #2.
λ_{xi}, $\lambda_{x\theta}$, λ_x	*x*-direction stretch compliances of member #*i*, of contribution due to the bending rotation of members, of the sandwich structure.
λ_{zi}, λ_z	Through-thickness compliances of member #*i*, of the sandwich structure.
μ, μ^*	Parameters that describe the differential bending compliances of members #2 and #1.
θ_i	Rotation of the centroid axis of member #*i* due to bending.
$\phi_3, \overline{\varphi}_i$	Rotation of member #3, average rotation of the cross-section of member #*i* w.r.t to their respective centroid axis due to shearing.
ΔT	Temperature change.

Mathematical symbols

${\alpha_z}^4$	$= \frac{\overline{D}_e}{4\lambda_z}$
β^2	$= \frac{\lambda_x}{\kappa_s}$
C_s	$= \frac{\mu^* \alpha_{21} \Delta T}{\lambda_z \kappa_{s3}\left(4\alpha_z^4 + \beta^4\right)}$
D_i	$= \frac{E_i h_i}{12}$
$\overline{D}_i$	$= \frac{1}{D_i}$
$\overline{D}_e$	$= \overline{D}_1 + \overline{D}_2$
μ	$= \frac{1}{2}\left(\frac{h_2}{D_2} - \frac{h_1}{D_1}\right)$
μ^*	$= \frac{1}{2}\left(\frac{h_2 + h_3}{D_2} - \frac{h_1 + h_3}{D_1}\right)$
κ_s	$= \kappa_{s1} + \kappa_{s2} + \kappa_{s3}$
κ_{si}	$\approx \frac{h_i}{8G_i}$
κ_{s3}	$= \frac{h_3}{G_3}$
λ_x	$= \lambda_{x1} + \lambda_{x2} + \lambda_{x\theta}$
λ_{xi}	$\approx \frac{1}{E_i h_i}$
$\lambda_{x\theta}$	$= \frac{1}{4}\left(\frac{{h_1}^2 + h_1 h_3}{D_1} + \frac{{h_2}^2 + h_2 h_3}{D_2}\right)$
λ_z	$= \lambda_{z1} + \lambda_{z2} + \lambda_3$
λ_{zi}	$\approx \frac{3h_i}{8E_i}$
λ_{z3}	$= \frac{h_3}{E_3}$

Note: The above formulae involving E_i are for plane stress; substitutes E_i with $E_i' = E_i/(1-{\nu_i}^2)$ for plane strain and $E_i' = E_i/(1-\nu_i)$ for spherical bending.

at where it is most critical – the free edge of the assembly. In contrast to the assumed uniform transverse stress along the thickness of the bonding layer, the theory of elasticity suggests that the transverse stress in a highly compliant bonding layer is made up of a uniform component and a linear component, with the magnitude of the latter significantly larger than the former except for severely asymmetric sandwich structures [40]. The magnitude of the linear component is directly proportional to the gradient of the interfacial shear stress; it is therefore essential that the interfacial shear stress is accurately modelled at and near the free edge where it acquires the highest magnitude of gradient. This calls for the enforcement of nil-shear-stress condition at the free edge. However, the bulk of the strength-of-material solutions did not enforce the nil shear stress condition at the free edge leading to an over estimation of the maximum magnitude of shear stress in the assembly while grossly underestimating the magnitude of the peeling stress at the free edge of the assembly. Understandably, it is unreliable to use these closed-form solutions for design analysis.

Suhir [28] and Ru [36] have tried to enforce the “nil shear stress” condition in a bi-material structure experiencing mismatched thermal expansion. Modelling the shear and stretch responses between the centroid axes of the two material as distribution of infinitesimal springs of first order Winkler model, Suhir [28] arrived at a sixth-order differential equation for the interfacial shear stress. Following the approach of Suhir but modelling the shear compliance between the centroid axes using a higher order Winkler model while assuming absolute stiffness in stretching, Ru [36] obtained a fourth-order differential equation for the interfacial shear stress. The condition of nil interfacial shear stress at the free edge of the structure was enforced as one of the three and two anti-symmetric boundary conditions, respectively. Unfortunately, both solutions have failed to model the expected singular nature of the interfacial peeling stress at the free edge of the bi-material structure.

In this manuscript, the interfacial shear stress in symmetric or mildly asymmetric sandwich structures due to differential thermal expansion and/or differential free-edge stretching is modelled using a second order differential equation and the nil stress condition at the free edge is enforced using a high power exponential function leading to concise yet accurate closed-form solutions; the interfacial peeling stress is modelled as the sum of a mean and an amplitude of variation of the transverse normal stress in the bonding layer, the latter is linearly proportional to the gradient of the interfacial shear stress. Following validation with the finite element analysis of these concise solutions, robust design analysis for debonding, fracturing, and out-of-plane deformation of sandwich structures are then performed.

2. The fundamental equations

Fig. 1 shows the elemental representations of a sandwich structure, wherein the structural members #1 and #2 are modelled as beam elements that have advanced descriptions for shear compliance. The bonding layer, member #3, is modelled as a two-dimensional elastic body that has negligible stiffness in the *x*-direction. The assumption leads to a constant distribution of shear