International Journal of Mechanical Sciences 117 (2016) 16–29

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Dynamic analysis of rectangular thin plates of arbitrary boundary conditions under moving loads

Qinghua Song [a,b,*], Jiahao Shi [a,b], Zhanqiang Liu [a,b], Yi Wan [a,b]

[a] Key Laboratory of High Efficiency and Clean Mechanical Manufacture, Ministry of Education, Jinan, People's Republic of China
[b] School of Mechanical Engineering, Shandong University, Jinan, People's Republic of China

ARTICLE INFO

Article history:
Received 22 December 2015
Received in revised form
17 May 2016
Accepted 3 August 2016
Available online 4 August 2016

Keywords:
Kirchhoff plate
Moving load
Inertia effect
Rayleigh–Ritz method
Courant's penalty method
Differential quadrature method

ABSTRACT

A comprehensive method is proposed to predict the dynamic behaviors of flat plate of arbitrary boundary conditions subjected to moving loads, based on Kirchhoff plate theory. The governing equations of motion are derived using the Lagrange equation. Rayleigh–Ritz method is employed and extended to treat the spatial partial derivatives. Different with conventional Rayleigh–Ritz solutions, the admissible functions adopted here integrate the advantages of both polynomials and trigonometric functions, which just satisfy a totally unconstrained condition, and Courant's penalty method is used to handle constraints. Differential quadrature method is used for discretization of temporal derivatives. The results show that the presented method is very reliable and efficient, and its convergence and accuracy are also better compared to finite element method. Moreover, the method is good for dealing with the boundary conditions due to employing suitable admissible functions. To illustrate this, the method presented evaluates the dynamic response of a plate example, whose three edges are usual constrains, and the fourth edge connects to a real spring with arbitrary length and stiffness value.

1. Introduction

Dynamic behaviors of flat plate under moving external loads are essential problems in structural dynamic field, which are commonly countered in engineering, such as bridges and roads, space vehicles, submarines and mechanical engineering. So many research works for this aspect have been conducted in past decades. Ouyang [1] has summarized a variety of engineering problems which are associated with the dynamics of structures under moving loads. Fryba [2] performed one of the first and most important studies on the moving load problem with several analytical solution methods. Here, a concise review of research studies related with dynamic problem of plate subjected to moving loads is carried out in the following section.

The governing equation of motion for the vibration problem of plate subjected to moving loads is partial differential equation. Hence, eigenfunction and integral scheme are commonly employed to treat the special and temporal derivatives, respectively. Gbadeyan et al. [3] presented a versatile solution technique based on modified generalized finite integral transforms and the modified Struble's method. Takabataka [4] considered the discontinuous variation of bending stiffness and mass of the plate with variation of the thickness using a characteristic function and presented an analytical method for rectangular plates under moving loads. Huang et al. [5] developed a procedure incorporating the finite strip method to treat the response of rectangular plate on elastic foundation subjected to a moving point loads. Shadnam et al. [6] presented a method to treat the response of rectangular plate under moving mass and force by transforming the governing equation into a series of eigenfunctions. Ghazvini et al. [7] introduced a robust computational approach to perform the transverse vibration of a thin rectangular plate of varying thickness under a traveling mass using eigenfunction expansion method. Nikkhoo et al. [8] proposed a semi-analytical model to study the response of a thin rectangular plate subjected to series of moving inertial loads by using eigenfunction expansion method. Gbadeyan et al. [9] investigated the elastodynamic response of a rectangular Mindlin plate subjected to a distributed moving mass by using a finite difference algorithm to transform the differential equation into a set of linear algebraic equations. Amiri et al. [10], based on first-order shear deformation plate theory, studied the response of a Mindlin elastic plate under a moving mass by using direct separation of variable and eigenfunction expansion method. Eftekhari et al. [11] presented a combined application of Ritz method, differential quadrature method and integral quadrature method to conduct the vibration response of rectangular plate subjected to accelerated traveling masses. In this paper, the Ritz method with beam eigenfunctions is used to discretize the spatial partial derivatives, and the differential quadrature method and

* Corresponding author at: Key Laboratory of High Efficiency and Clean Mechanical Manufacture, Ministry of Education, School of Mechanical Engineering, Shandong University, Jinan, People's Republic of China.
E-mail addresses: ssinghua@sdu.edu.cn (Q. Song), melius@sdu.edu.cn (Z. Liu).

http://dx.doi.org/10.1016/j.ijmecsci.2016.08.005
0020-7403/

integral quadrature method are employed to analogize the resultant system of partial differential equations, then the Newmark time integration scheme is used to solve the ordinary differential equations.

Finite element method, which is one of the most versatile methods to solve the spatial problem, is often applied in the vibration problem of plate under moving loads. Wu [12] investigated vibration of a rectangular plate under moving force along a circular path using finite element method associated with equivalent forces (and moments). Wu [13,14] presented a moving (distributed) mass element to perform the dynamic analysis of an inclined plate under moving (distributed) loads using finite element method. Esen [15] presented an equivalent finite element to analyze the transverse vibration of the plate under a moving point mass.

This paper presents a comprehensive method to treat the dynamic problem of thin plate with arbitrary boundary conditions subjected to moving loads (force and mass) based on thin plate theory. The governing equations of motion are derived using the Lagrange equation. The updated Rayleigh–Ritz method associated with Courant's penalty method is employed to deal with the spatial partial derivatives. The admissible functions adopted here just satisfy a totally unconstrained condition. Then the differential quadrature method is used for discretization of temporal derivatives.

2. Dynamic model of flat plate with moving loads

As shown in Fig. 1, a flat plate of length L, width W and thickness h, supporting a mass M traveling at a velocity v_M, is the physical system analyzed here. Without loss of generality, it is assumed that the moving path of the mass is parallel to the o-x axis, that is, the locations of mass can be written as (x_M, y_M), where $x_M = v_M t$, y_M=constant, as shown in Fig. 1. In this study it is assumed that deflections are small, and the Kirchhoff plate theory provided by [16] is valid. Accordingly, in the case that the system is an isotropic plate and there is no damping in the plate and loading system, the differential equation of the motion of the plate is [2,16]:

$$D\left[\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right] + \rho h \frac{\partial^2 w}{\partial t^2} = F(x, y, t) \tag{1}$$

where $D = Eh^3/[12(1-\mu^2)]$ is the flexural rigidity and E, μ and ρ are Young's modulus of elasticity, Poisson's ratio and density of plate, respectively. $F(x,y,t)$ is moving load. w is lateral deflection of plate.

In order to deal with arbitrary boundary conditions of plate, here the Rayleigh–Ritz solutions are directly applied into both the energy terms of deflections of plate and the energy terms due to boundary conditions, and then the Lagrange equation is employed to obtain the governing equation of motion of plate including boundary conditions. Therefore, the Lagrange of the present problem is firstly deduced as

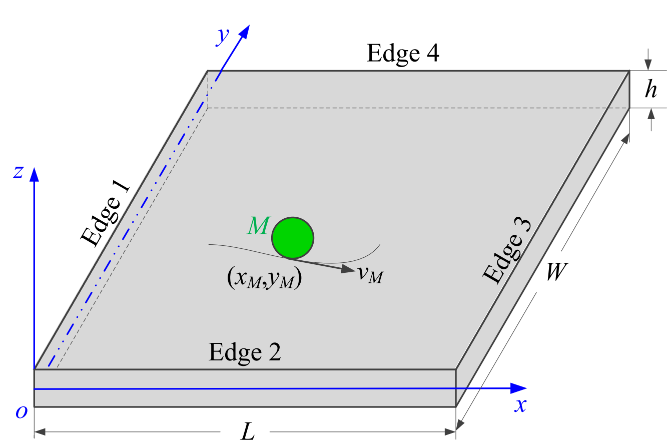

Fig. 1. Moving loads traveling on the plate with arbitrary trajectory.

follows. To exactly describe the motion of the plate, three displacement components, namely w, u, and v, which represent the displacement along o-z axis, o-x axis and o-y axis, respectively, are necessary. Based on the Kirchhoff plate theory, the horizontal and vertical displacements of the plate are expressed as

$$\begin{aligned} u(x, y, z, t) &= u_0(x, y, t) - z w_{0,x}(x, y, t) \\ v(x, y, z, t) &= v_0(x, y, t) - z w_{0,y}(x, y, t) \\ w(x, y, z, t) &= w_0(x, y, t) \end{aligned} \tag{2}$$

where u_0, v_0 and w_0 are the horizontal and vertical displacements of the plate in the middle plane (x–y plane), and $(\cdot)_x$ and $(\cdot)_y$ represent the partial differentiations with respect to x and y, respectively. The stain–displacement relationships are expressed as

$$\varepsilon_x = u_{0,x} - z w_{0,xx}, \quad \varepsilon_y = v_{0,y} - z w_{0,yy}, \quad 2\varepsilon_{xy} = u_{0,y} + v_{0,x} - 2z w_{0,xy} \tag{3}$$

where ε_x, ε_y are normal strain, and ε_{xy} is shear strain. $(\cdot)_{xx}$, $(\cdot)_{yy}$, $(\cdot)_{xy}$ represent the second-order partial derivatives with respect to x and y, respectively. The corresponding constitutive relations are

$$\sigma_x = \frac{E}{1-\mu^2}(\varepsilon_x + \mu\varepsilon_y), \quad \sigma_y = \frac{E}{1-\mu^2}(\varepsilon_y + \mu\varepsilon_x), \quad \sigma_{xy} = \frac{E}{2(1+\mu)}\varepsilon_{xy} \tag{4}$$

where σ_x, σ_y are normal stress, and σ_{xy} is shear stress. μ and E represent Poisson ratio and elasticity modulus of plate.

Therefore, the strain energy (V) and kinetic energy (T) of the plate are

$$V = \frac{1}{2}\int_0^L \int_0^W (\sigma_x \varepsilon_x + 2\sigma_{xy}\varepsilon_{xy} + \sigma_y \varepsilon_y) dxdy \tag{5}$$

$$T = \frac{1}{2}\int_0^L \int_0^W \rho h(\dot{w}^2 + \dot{u}^2 + \dot{v}^2) dxdy \tag{6}$$

Substituting Eqs. (2)–(4) in Eqs. (5) and (6) leads to:

$$V = \frac{1}{2}\int_0^L \int_0^W \left\{ \begin{aligned} &\frac{Eh}{1-\mu^2}\left[u_{0,x}^2 + v_{0,y}^2 + \frac{1-\mu}{2}(u_{0,y} + v_{0,x})^2 + 2\mu u_{0,x} v_{0,y}\right] \\ &+ \frac{Eh^3}{12(1-\mu^2)}\left[w_{0,xx}^2 + w_{0,yy}^2 + 2(1-\mu) w_{0,xy}^2 + 2\mu w_{0,xx} w_{0,yy}\right] \end{aligned} \right\} dxdy \tag{7}$$

$$T = \frac{1}{2}\int_0^L \int_0^W \left[\rho h(\dot{w}_0^2 + \dot{u}_0^2 + \dot{v}_0^2) + \frac{\rho h^3}{12}(\dot{w}_{0,x}^2 + \dot{w}_{0,y}^2)\right] dxdy \tag{8}$$

Additionally, the work due to the weight and inertia of the moving mass, which is expressed as

$$\begin{aligned} W &= -\int_0^L \int_0^W \left[Mg + M\ddot{w}_0(x_M, y_M, t)\right]\delta(x - x_M)\delta(y - y_M) w_0(x, y, t) dxdy \\ &= -M\left[g + \ddot{w}_0(x_M, y_M, t)\right] w_0(x_M, y_M, t) \end{aligned} \tag{9}$$

where $\delta(.)$ denotes the Dirac delta-function. When the effect of inertia of the moving load is neglected, the problem is referred to a moving force problem, and otherwise is referred to a moving mass problem.

Based on previous analysis, the Lagrangian of the present problem is

$$L = T - V - W \tag{10}$$

3. Rayleigh–Ritz analyses

From Eqs. (7)–(9), it can be seen that only three unknown parameters are needed to obtain the total energy of system,