International Journal of Mechanical Sciences 115-116 (2016) 12–23

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Attenuation zones of initially stressed periodic Mindlin plates on an elastic foundation

Xinnan Liu [a], Zhifei Shi [a,*], Y.L. Mo [b]

[a] School of Civil Engineering, Beijing Jiaotong University, Beijing 100044, PR China
[b] Department of Civil and Environmental Engineering, University of Houston, Houston, TX 77004, USA

ARTICLE INFO

Article history:
Received 30 September 2015
Received in revised form
8 May 2016
Accepted 13 June 2016
Available online 15 June 2016

Keywords:
Attenuation zones
Initial stress
Periodic plates
Elastic foundation
Weak form quadrature element method

ABSTRACT

A new numerical approach based on the weak form quadrature element method is proposed to study attenuation zones of initially stressed periodic Mindlin plates on an elastic foundation. The proposed method is validated by the results available in the previous studies of initially stressed homogeneous plates on elastic foundations. A comprehensive parametric study is conducted to highlight the effects of initial stress, geometric parameters and elastic foundation on the attenuation zones. The results show that the compressive initial stress shifts the attenuation zones to lower frequencies and enhances the attenuation of waves in the attenuation zones, while the tensile initial stress shifts the attenuation zones to higher frequencies and weakens the attenuation of waves in the attenuation zones. The elastic foundation shifts the attenuation zones to higher frequencies and narrows the width of the attenuation zones. In addition, wave propagation and attenuation in a periodic Mindlin plate with finite unit cells on an elastic foundation are examined. The results achieved in this paper are very useful for the design and application of periodic plates in vibration reduction in most of engineering fields.

1. Introduction

Periodic structures or composites, whose material properties or geometric configurations are spatially periodic, have received much attention in the past two decades [1,2]. Periodic structures possess unique dynamic characteristic of frequency attenuation zones (AZs). Waves with frequencies in the AZs cannot propagate through the periodic structures. Hence, the AZ characteristic makes periodic structures a potential candidate in many engineering applications, such as mechanical filters, acoustic suppression and seismic isolation [3–7].

Recently, various periodic plates that can be considered as a type of 2D periodic structures are attracting attention. Integrating the finite element method (FEM) with an optimization algorithm, El-Sabbagh et al. [8] studied the topology optimization of periodic Mindlin plates to maximize the fundamental frequency of the plates, where the periodicity is introduced by spatially periodic thickness distribution. Using the FEM, Spadoni et al. [9] studied vibration control in plates by mounting periodic array of shunted piezo-patches on the plates. In spite of the wide use of traditional FEM in the analyses of periodic plate structures, it is of interest to develop new methods to study the AZs. Using spectral element method (SEM), Wu et al. [10] investigated the AZs of periodic thin plates, in which it is shown that the larger the differences of the material properties of the components in the periodic plates, the greater vibration attenuation in AZs can be obtained. Subsequently, Wu et al. [11] extended the SEM to study the AZs of periodic sandwich plates with corrugated cores. In these studies [10,11], it is found that the spectral element method gives more accurate results with fewer elements when compared to the traditional FEM. More recently, the differential quadrature element method (DQEM) is introduced in the study of periodic plates. Xiang et al. [12] studied the AZs for in-plane waves in a plate with periodic piezoelectric patches. In their study [12], the DQEM is proved with high accuracy and fast convergence in calculating the AZs. Further, Cheng et al. [13] extended the DQEM in the study of the AZs for flexural waves in a plate with periodic piezoelectric patches, in which the detailed solution procedure with the DQEM is presented.

Two important factors should be considered in the application of periodic structures in engineering. The first one is the soil-structure interaction, which is of both theoretical and practical interest in the analyses of highway pavement, railroad structures and foundations of buildings. To simplify the soil-structure interaction, several elastic foundation models have been proposed, such as the Winkler model (one-parameter foundation) and the Pasternak model (two-parameter foundation) [14,15]. Investigations on the effect of elastic foundation on the AZs of periodic

* Corresponding author.
E-mail address: zfshi178@bjtu.edu.cn (Z. Shi).

http://dx.doi.org/10.1016/j.ijmecsci.2016.06.010
0020-7403/

structures are limited. Xiang and Shi [16] and Yu et al. [17] studied the flexural AZs of various types of periodic beams on elastic foundation. To the knowledge of the present authors, flexural AZs of periodic plates on elastic foundation have not been studied. The second factor is the initial stress. Due to the large difference between material properties of the components of periodic structures, the initial stress is unavoidable during the manufacturing process or the service life [18,19]. It has been shown that initial stress can significantly alter the AZs of periodic structures [20,21]. However, the effect of initial stress on flexural AZs of periodic plates has not been studied.

In this paper, a new method based on the weak form quadrature element method (WFQEM) is developed to study the AZs for flexural waves in initially stressed periodic Mindlin plates on elastic foundation. The WFQEM is a newly developed numerical method by adopting the energy principle and the DQEM, in which neither trial solutions nor meshes are required. Due to its high accuracy, the WFQEM has been applied to the static and dynamic analyses of plane stress/strain problems, beams and plates [22–24]. Although the WFQEM is well established, research is needed in order to extend the WFQEM to study the periodic structures. First, as for the solution procedure for the AZs and the attenuation coefficients in periodic plates with infinite unit cells, the Bloch analysis method should be incorporated with WFQEM to consider the periodicity of the periodic plates. Second, as for the dynamic responses of periodic plates with finite unit cells, time-domain integration methods should be combined with the WFQEM. The remaining sections are organized as follows: The proposed computational method is developed in Section 2. In Section 3, the accuracy of the proposed method is verified by the results available in the previous studies of homogeneous plates on elastic foundations. In Section 4, the AZs of initially stressed periodic Mindlin plates on an elastic foundation are investigated. In Section 5, the frequency-domain and time-domain responses of periodic Mindlin plates with finite unit cells on an elastic foundation are analyzed to verify the results in Section 4. Finally, some conclusions are provided in Section 6.

2. Proposed computational method

Consider an initially stressed periodic plate resting on a Pasternak foundation, where thickness of the periodic plate is h, side length of the inclusion is l, and periodic constant (i.e., side length of the typical unit cell) is a, as shown in Fig. 1. The Pasternak foundation is characterized by two parameters, i.e., the Winkler modulus k_w and the shear layer modulus k_s. In the present paper, it is assumed that: (i) the materials of both the inclusion and the matrix are isotropic and linearly elastic; (ii) the interfaces between the inclusion and the matrix are perfectly bonded; and (iii) the damping of both the plate and Pasternak foundation are negligible.

2.1. Governing equations

The typical unit cell is decomposed into nine subdomains, as shown in Fig. 2(a). It should be noted that mapping technique is not necessarily required for the configuration in the present paper where all the subdomains are rectangles [25]. For more general configurations where the subdomains are irregular, mapping technique has to be introduced [26,27]. Without loss of generality, mapping technique is adopted in the present paper.

An arbitrary subdomain e in the (x, y) coordinate system can be mapped into a normalized square in the (ξ, η) coordinate system as shown in Fig. 2(b) by using the following transformation:

$$\begin{cases} x = x(\xi, \eta) = \sum_{i=1}^{n} S_i(\xi, \eta)x_i \\ y = y(\xi, \eta) = \sum_{i=1}^{n} S_i(\xi, \eta)y_i \end{cases} \quad -1 \le \xi, \eta \le 1, \tag{1}$$

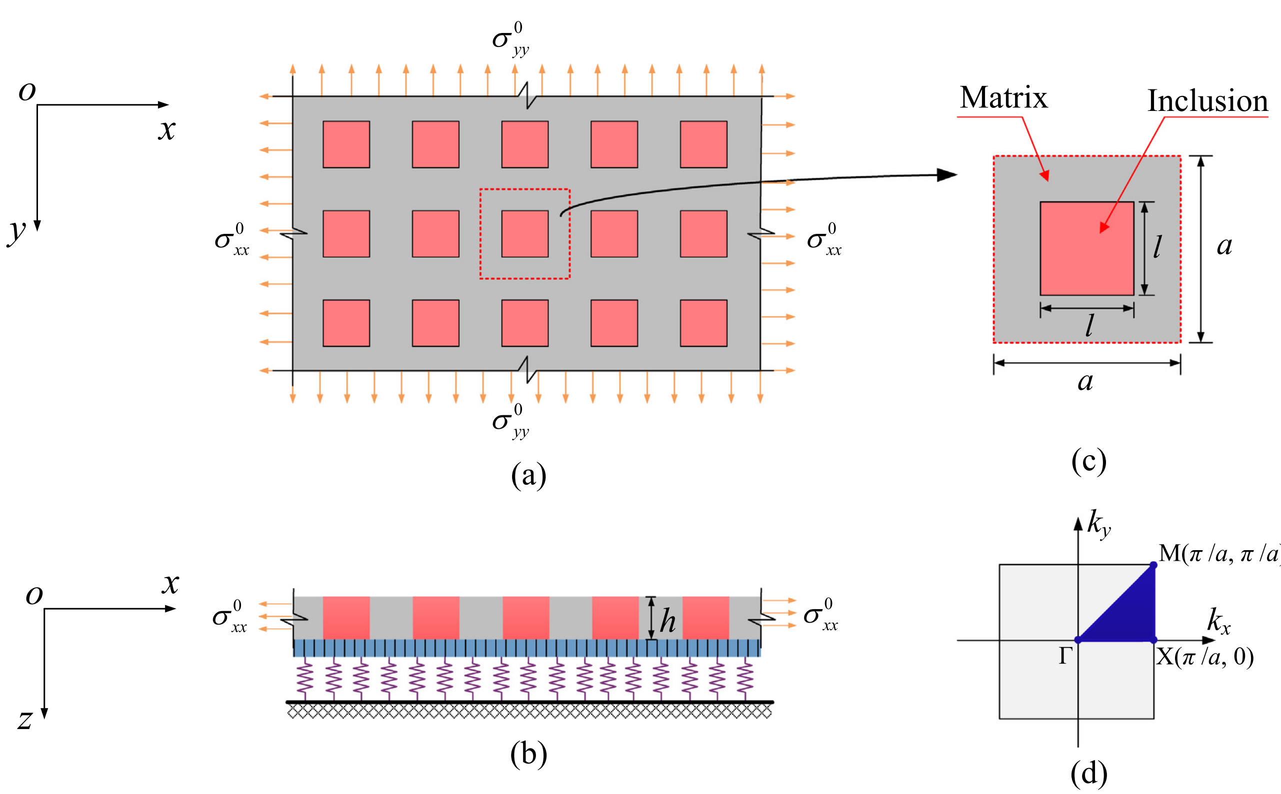

Fig. 1. (a) Top view and (b) side view of an initially stressed periodic Mindlin plate on Pasternak foundation, (c) a typical unit cell and (d) the corresponding first Brillouin zone.