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An incremental approach for springback analysis of elasto-plastic beam undergoing contact driven large deflection

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ABSTRACT

In this article the solution methodology for a beam on a vee-die undergoing large elasto-plastic deflection along with nonlinear contact development with the die is discussed. A bi-linear stress strain material model is converted into an incremental moment- curvature based constitutive law to ease formulation. The one dimensional governing equation obtained, is highly nonlinear owing to material and geometry and involve boundary condition change. The entire problem is solved in three steps: solving an end loaded cantilever under non-conservative force, followed by choosing the solutions which satisfy the configurational constraint and finally reanalyzing the contact data for springback analysis. The end loaded cantilever problem is solved by an incremental procedure coupled with Runge–Kutta fourth-order explicit initial value solver. Suitable normalization of the pertinent variables of the governing equation paved the way to identify dependence of the responses on a unique non-dimensional parameter. The presented methodology doesn't involve large matrix inversion and so is computationally economic. It may be used in sheet metal manufacturing control facilities to predict springback and reduce the expensive experimental iterations.

1. Introduction

Since the time Euler introduced the ‘elastica’ theory approximately 200 years ago, the study of large deflection of flexible elastic beams has been an ever evolving research area, see [14] . In the last few decades the minimum weight criteria for the aerospace industry have led to renewed interest in this area of research, as noted by [5]. Incorporation of elasto-plastic nonlinearity in the formulation have led to its application in the sheet metal forming industry also, as seen in [16]. To avoid expensive trial and error experimental procedure and computationally intensive as well as exhaustive FEM analysis, a simple algorithm to render fair estimate of springback and load-deformation response is the subject matter of the present article.

Analytical solutions to large deflection of elastic beam exist in the form of evaluation of elliptic integrals, Jacobian integrals etc. [2] contains a short review of analytical methods and pertinent references.

The analytical methods are explicit in terms of angle or slope of the elastic line and are implicit in load and deflection. To obtain load or displacement solution, iterations are required. For rendering explicit solution, based on displacement and load, several semi-analytical techniques are formulated. Among these, Adomian decomposition and homotopy perturbation methods are popular, they can be found in [8,1] and more recently in [3].

Semi-analytical methods are more efficient in solving problems with conservative material nonlinearity. Plasticity and friction induced deformations are inherently load-deflection path dependent in nature. Purely numerical methods like FEM can be used for such problems involving history dependent material property or follower type load. However owing to computational intensive demand of FEM, other non-FEM based numerical techniques are of research interest. Some of the relevant references can be found in the following but not limited to these are: [23,21,4,10].

In a typical sheet metal air bending process, a thin sheet is placed horizontally over a vee-die and a punch descends vertically to deform it plastically, as shown in Fig. 1. With release of the punch the sheet retracts back partially owing to elastic deformation. This springback phenomenon is undesirable and hence an accurate prediction is necessary for satisfactory over-bending of the sheet to produce desired bend angle. The entire problem is highly complicated due to nonlinearity in geometry and material property. Detailed modeling employing FEM, enables a realistic simulation of the entire deformation phenomenon, showing the dependence on process variables. Several references on FEM simulation are available, see for example [13,17,20,11,18] (and the references therein). However, quite often a simple mechanics of materials based model is required for concept design and forming control algorithm, see [16]. For forming of long sheets or beams,

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Nomenclature

x	the horizontal coordinate as shown in Fig. 2.
y	the vertical coordinate as shown in Fig. 2.
E	Young's modulus
m	ratio of tangent modulus to Young's modulus
I	area moment of inertia of beam cross section about neutral axis
l	length of beam as shown in Fig. 2.
δ	vertical displacement of wedge tip point, i.e point B in Fig . Fig 4
h, q	vertical and horizontal coordinates of point of contact of beam to wedge respectively, i.e point A of Fig. 4.
a	length of contact as shown in Fig. 4.
ϕ	angle in radians the tangent at any point on the deformed beam makes with the x axis , as shown in Fig. 2.
ψ	angle in radians the tangent at the free end of the deformed beam makes with the x axis , as shown in Fig. 2.
s	coordinate of any point on the deformed beam measured from fixed end along the beam as shown in Fig. 2.
L	horizontal distance of vertical edge of wedge to fixed end as shown in Fig. 4.
θ	the wedge angle in radians as shown in Fig. 4.
γ	the vee-die angle as shown in Fig. 5.
$\bar{s}, \bar{x}, \bar{y}$	respective quantities normalized with respect to L, i.e $\bar{(.)} = \frac{(.)}{L}$
P	the follower load at the free end of cantilever as shown in Fig. 2.
$\bar{P}$	the normalized follower load $=\frac{PL^2}{EI}$
σ_y	initial yield stress of the material in uni-axial tension
ϵ_y	initial yield strain of the material in uni-axial tension
$2b$	thickness of rectangular beam cross-section
w	width of rectangular beam cross section
κ	curvature of deformed beam$=\frac{\partial\phi}{\partial s}$
κ_{y0}	initial yield curvature of deformed beam $=\frac{\sigma_y}{Eb}$
κ^*	yield normalized curvature $=\frac{\kappa}{\kappa_{y0}}$
$\bar{\kappa}$	length normalized curvature$=\kappa L$
M	bending moment at any point on the cantilever
M_{y0}	initial yield bending moment of beam cross section $=\frac{2wb^2E\epsilon_y}{3}$
M^*	yield normalized moment $=\frac{M}{M_{y0}}$
D	tangent modulus of flexural rigidity : slope of $M-\kappa$ curve
$\bar{D}$	normalized D used in governing equation $=\frac{D}{EI}$
D^*	normalized D used in constitutive law, $=\frac{dM^*}{d\kappa^*}$
$(.)'$	derivative operator $=\frac{\partial(.)}{\partial\bar{s}}$
$\Delta(.)$	incremental operator $=\frac{\partial(.)}{\partial\psi}\Delta\psi$
RK4	Runge–Kutta classical explicit method for solving initial value problems
ζ	elastica parameter $=\kappa_{y0}L$
η	normalized springback, defined in Eq. (28).

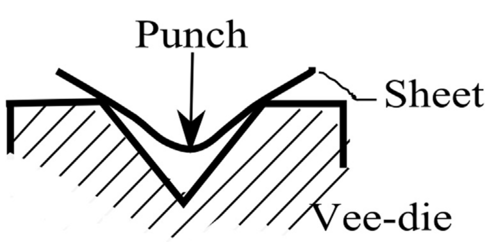

Fig. 1. Schematic of vee-die bending.

where the bent curvature is more than three times of that of sheet thickness, see [9], such models are fairly accurate. These simplified models owing to computational inexpensiveness, may be used for automation. These simplified models additionally aid in interpreting a full field finite element analysis result and thereby enhances the understanding of the designer and manufacturer, see [9].

One of the benchmark works in the field of analytic prediction of springback in sheet metal bending is provided by [7]. He considered pure bending of an elasticperfectly plastic beam. [12] obtained analytical springback expressions of beam and thin sheet made of hardening materials undergoing pure bending. By considering a rigid-linearly hardening moment-curvature material model, [22] predicted springback in press brake sheet bending. These analytic solutions are based on monotonic material models, and hence are not suitable for situations where incremental law is required. Recently, [6] obtained vee-bending springback graphics depending on various process parameters by conducting different experiments and considering power law kind of stress–strain relationship. In the analytic field, [24] obtained the solution of springback of sheet metal undergoing vee bending, using Hill's quadratic yield function and an exponential hardening rule in the pure bending framework. In very recent years, within pure bending framework [25] and [26] predicted final curvature of sheets, by considering monotonic power law hardening relationship. For a comprehensive review of the springback of mono-layer and bi-layer sheets, [15] may serve as a good reference.

The existing literature on mechanics of materials based models, mostly consider uniform curvature over the deformed span of the beam with monotonic material models. On the other hand, the incremental constitutive law based approaches are largely based on FEM. The objective of the present work is to present a non-FEM numerical method, based on mechanics of materials to explicitly solve the vee-die bending of a thin beam with the more realistic non-uniform curvature situation considering an incremental material model. In this study, an isotropic hardening constitutive law is presented in incremental form. The approach presented here is such that, other hardening laws like kinematic or mixed can be easily integrated into it making the approach generic for cyclic loading.

It is imperative that a cantilever problem is equivalent to solving vee-die bending of a beam owing to symmetry in loading and support. Thus we consider a horizontal cantilever undergoing large plastic deflection due to the contact driven action of a wedge descending vertically downward, as shown in Fig. 2. For an end loaded cantilever beam, kinematic, constitutive and equilibrium equations are combined to obtain the governing differential equation, with slope angle as the primary dependent variable. The obtained nonlinear differential equation is linearized and subsequently solved for every (pseudo) time step using Runge–Kutta fourth-order (RK4) initial value problem solver. The process is repeated for various wedge angles, lengths of beam and material properties. Only those solutions that satisfy the pertinent configuration constraint are employed to create a feasible solution set for load-displacement and contact responses. The feasible data are subsequently reanalyzed to obtain springback responses under contact conditions depending on various process parameters.

The method presented here is simple to formulate, easy to