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Axisymmetric conducting indenter on a functionally graded piezoelectric coating

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ABSTRACT

The indentation response of a functionally graded piezoelectric material (FGPM) coating-substrate system under an axisymmetric conducting indenter is investigated. The material properties of the FGPM coating vary in an exponential form along the thickness direction. The coupled singular integral equations are derived to describe the contact problem of the FGPM coating-substrate system under an axisymmetric conducting indenter. The singular integral equations are solved numerically to achieve the electrical and mechanical responses of the FGPM coating for different indenters, material properties and substrates. The numerical results show the effects of the conductivity of the indenter on the distribution of the contact stress and the electric charge.

1. Introduction

According to the concept of functionally graded materials (FGM) [1], functionally graded piezoelectric materials (FGPM) [2] are developed to improve the performance of the conventional piezoelectric materials (PM) which are widely used as sensors, actuators and transducers. The advantages of the FGPM have been highlighted by many researchers [3]. They found that the FGPM can be applied to reduce the mechanical stresses, improve the stress distributions, enlarge the output displacements and increase the bonding strength.

The indentation problems of piezoelectric materials have been analyzed as an important topic in the past few years. The two-dimensional (2D) analytical solution for piezoelectric materials under a point force and a point charge was obtained with the technique of Fourier transforms [4]. Ding [5] solved the problem of a point force and a point charge acting on a piezoelectric half-space. The three-dimensional (3D) contact problem for piezoelectric materials was also studied by Ding et al. [6]. A general theory for the indentation of piezoelectric materials under an axisymmetric indenter was developed by Giannakopoulos and Suresh [7]. The theoretical and numerical results for three kinds of the indenter on a piezoelectric half-space were obtained in their work. Sridhar et al. [8] conducted an indentation experiment to study the mechanical and electrical responses of piezoelectric materials indented by a conical indenter. They demonstrated that the indentation experiment is a convenient tool for the characterization of the material properties. By using the potential theory, Chen [9,10] investigated the contact problem for a transversely isotropic piezoelectric material indented by an axisymmetric indenter. The mechanical and electrical fields for a piezoelectric layer with a circular surface electrode and a circular indenter were investigated by Wang et al. [11,12]. The frictionless indentation responses of a piezoelectric film attached to a rigid substrate system under axisymmetric indenters were considered by Wang et al. [13] and Wang and Chen [14]. In their research works, the closed-form solutions were obtained by using the technique of the Hankel transform. Zhou and Li [15] studied the frictional sliding contact problem of magneto-electro-elastic materials under a rigid punch by using the technique of singular integral equations. Recently, the 2D contact problems of an FGPM layer coated on a half-plane were treated by Ke et al. [16–18]. They assumed that the material properties of the FGPM coating vary in an exponential form along the thickness direction and solved the problem by using the technique of singular integral equations. However, to the author's knowledge, the 3D axisymmetric contact problem of the FGPM coating attached to a half-space has not been presented in literature.

Motivated by the previous research works on contact problems of the piezoelectric materials, the electrical and mechanical responses of the FGPM coating for 3D axisymmetric conducting indenters are investigated in this paper. The material properties are

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assumed to vary exponentially along the thickness direction. The coupled singular integral equations, which describe the 3D axisymmetric contact problem of the FGPM coating, are derived by using the method of the Hankel integral transform. The contact stress and the electric charge distributions for a cylindrical indenter and a spherical indenter are computed by solving the singular integral equations numerically. The effects of the material gradient on the electrical and mechanical responses for different indenters are analyzed. The numerical results for different piezoelectric substrates are also presented and discussed.

2. Derivation of the fundamental solution

Let us consider a functionally graded piezoelectric material (FGPM) coating of the thickness h_0 in the cylindrical coordinate system (r, θ, z), which is attached to a piezoelectric substrate, as shown in Fig. 1. Both the FGPM coating and the piezoelectric substrate are transversely isotropic. The substrate is treated as a half-space. The axisymmetric conducting indenter with the electric potential ϕ is loaded by an applied force P on the FGPM coating-substrate system to form the contact region whose radius is a. The poling direction is parallel to the z-axis. The material properties of the FGPM coating are assumed to have the following form:

$$\{c_{kl}(z), e_{kl}(z), \in_{kl}(z)\} = \{c_{kl0}, e_{kl0}, \in_{kk0}\}e^{\beta z}, \ (0 \le z \le h) \tag{1}$$

where c_{kl}, e_{kl} and $\in_{kl}$ are the elastic, piezoelectric and dielectric constants, respectively. In Eq. (1), β is the gradient index, and c_{kl0}, e_{kl0} and $\in_{kl0}$ are the material properties of the piezoelectric substrate.

The governing partial differential equations for the non-homogeneous medium are given by [19,20]

$$c_{110}\left(\frac{\partial^2 u_{rj}}{\partial r^2} + \frac{1}{r}\frac{\partial u_{rj}}{\partial r} - \frac{u_{rj}}{r^2}\right) + c_{440}\frac{\partial^2 u_{rj}}{\partial z^2} + (c_{130} + c_{440})\frac{\partial^2 u_{zj}}{\partial r \partial z} + (e_{310} + e_{150})\frac{\partial^2 \phi_j}{\partial r \partial z} + \beta\left\{c_{440}\left(\frac{\partial u_{rj}}{\partial z} + \frac{\partial u_{zj}}{\partial r}\right) + e_{150}\frac{\partial \phi_j}{\partial r}\right\} = 0, \tag{2a}$$

$$c_{440}\left(\frac{\partial^2 u_{zj}}{\partial r^2} + \frac{1}{r}\frac{\partial u_{zj}}{\partial r}\right) + c_{330}\frac{\partial^2 u_{zj}}{\partial z^2} + (c_{130} + c_{440})\frac{\partial}{\partial z}\left(\frac{\partial u_{rj}}{\partial r} + \frac{u_{rj}}{r}\right) + e_{150}\left(\frac{\partial^2 \phi_j}{\partial r^2} + \frac{1}{r}\frac{\partial \phi_j}{\partial r}\right) + e_{330}\frac{\partial^2 \phi_j}{\partial z^2} + \beta\left\{c_{130}\left(\frac{\partial u_{rj}}{\partial r} + \frac{u_{rj}}{r}\right) + c_{330}\frac{\partial u_{zj}}{\partial z} + e_{330}\frac{\partial \phi_j}{\partial z}\right\} = 0, \tag{2b}$$

$$e_{150}\left(\frac{\partial^2 u_{zj}}{\partial r^2} + \frac{1}{r}\frac{\partial u_{zj}}{\partial r}\right) + e_{330}\frac{\partial^2 u_{zj}}{\partial z^2} + (e_{150} + e_{310})\frac{\partial}{\partial z}\left(\frac{\partial u_{rj}}{\partial r} + \frac{u_{rj}}{r}\right) - \in_{110}\left(\frac{\partial^2 \phi_j}{\partial r^2} + \frac{1}{r}\frac{\partial \phi_j}{\partial r}\right) - \in_{330}\frac{\partial^2 \phi_j}{\partial z^2} + \beta\left\{e_{310}\left(\frac{\partial u_{rj}}{\partial r} + \frac{u_{rj}}{r}\right) + e_{330}\frac{\partial u_{zj}}{\partial z} - \in_{330}\frac{\partial \phi_j}{\partial z}\right\} = 0, \tag{2c}$$

where u_{rj} and u_{zj} are the radial and axial components of the displacement vector, ϕ_j is the electric potential, $j = 1$ refers to the FGPM coating and $j = 2$ with $\beta = 0$ to the piezoelectric half-space. Ueda [19] has given the solutions of the governing Eq. (2) for the axially symmetric problem of FGPM, so we can use these solutions to deal with the contact problem of FGPM.

In the Hankel-transformed domain, the solutions of the partial differential Eq. (2) for the FGPM coating ($j = 1$) and for the piezoelectric half-space ($j = 2$ with $\beta = 0$) are given in Appendix A.

The continuity conditions at the interface $z = 0$ can be written as

$$\sigma_{zz1}(r, 0) = \sigma_{zz2}(r, 0), \ \sigma_{zr1}(r, 0) = \sigma_{zr2}(r, 0), \ D_{z1}(r, 0) = D_{z2}(r, 0), \tag{3a,b,c}$$

$$u_{z1}(r, 0) = u_{z2}(r, 0), \ u_{r1}(r, 0) = u_{r2}(r, 0), \ \phi_1(r, 0) = \phi_2(r, 0). \tag{3d,e,f}$$

where $\sigma_{zzj}(r, z)$ and $\sigma_{zrj}(r, z)$ with $j = 1, 2$ are the normal stress and shear stress components, respectively, and $D_{zj}(r, z)(j = 1, 2)$ represents the electric displacement components.

Because the frictionless contact problem is considered, the mechanical boundary conditions along the coating surface $z = h_0$ are given as follows:

$$\sigma_{zz1}(r, h_0) = p(r), \ (0 \le r \le a), \ \sigma_{zz1}(r, h_0) = 0 (r > a), \ \sigma_{rz1}(r, h) = 0 \ (r > 0), \tag{4a,b,c}$$

where $p(r)$ is the unknown stress distribution and satisfies the following equilibrium condition

$$P = 2\pi \int_0^a p(t) t dt. \tag{5}$$

The electrical boundary conditions for a conducting indenter can be stated as

$$D_{z1}(r, h) = q(r) \ (0 \le r \le a), \ D_{z1}(r, h) = 0 \ (r > a), \tag{6a,b}$$

where $q(r)$ is the unknown electric charge distribution on the surface. Note that $q(r)$ is zero for an insulating indenter. The total electric charge Q can be obtained by

$$Q = 2\pi \int_0^a q(t) t dt. \tag{7}$$

In the Hankel-transformed domain, the continuity and boundary conditions in (3), (4) and (6) may be written as

$$T_1 A_1 = T_2 A_2, \tag{8}$$

$$S = T_3 A_1, \tag{9}$$

where

$$T_1 = \begin{bmatrix} \omega_{111}(s) & \omega_{121}(s) & \omega_{131}(s) & \omega_{141}(s) & \omega_{151}(s) & \omega_{161}(s) \\ \omega_{211}(s) & \omega_{221}(s) & \omega_{231}(s) & \omega_{241}(s) & \omega_{251}(s) & \omega_{261}(s) \\ \omega_{311}(s) & \omega_{321}(s) & \omega_{331}(s) & \omega_{341}(s) & \omega_{351}(s) & \omega_{361}(s) \\ \omega_{411}(s) & \omega_{421}(s) & \omega_{431}(s) & \omega_{441}(s) & \omega_{451}(s) & \omega_{461}(s) \\ \omega_{511}(s) & \omega_{521}(s) & \omega_{531}(s) & \omega_{541}(s) & \omega_{551}(s) & \omega_{561}(s) \\ -\omega_{611}(s) & -\omega_{621}(s) & -\omega_{631}(s) & -\omega_{641}(s) & -\omega_{651}(s) & -\omega_{661}(s) \end{bmatrix}, \tag{10a}$$

$$T_2 = \begin{bmatrix} \omega_{142}(s) & \omega_{152}(s) & \omega_{162}(s) \\ \omega_{242}(s) & \omega_{252}(s) & \omega_{262}(s) \\ \omega_{342}(s) & \omega_{352}(s) & \omega_{362}(s) \\ \omega_{442}(s) & \omega_{452}(s) & \omega_{462}(s) \\ \omega_{542}(s) & \omega_{552}(s) & \omega_{562}(s) \\ -\omega_{642}(s) & -\omega_{652}(s) & -\omega_{662}(s) \end{bmatrix}, \tag{10b}$$

$$A_1 = \begin{bmatrix} A_{11}(s) & A_{21}(s) & A_{31}(s) & A_{41}(s) & A_{51}(s) & A_{61}(s) \end{bmatrix}, \tag{10c}$$

$$A_2 = \begin{bmatrix} A_{42}(s) & A_{52}(s) & A_{62}(s) \end{bmatrix}, \tag{10d}$$

$$S = \begin{bmatrix} g_1(s) & 0 & g_2(s) \end{bmatrix}^T = g_1(s) S_1 + g_2(s) S_2, \tag{10e}$$