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Vibration analysis of rotating composite beams using polynomial based dimensional reduction method

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ABSTRACT

Free vibration analysis of rotating composite beams with arbitrary cross section is presented. The analysis is based on dimensional reduction method. Contrary to most of the previous studies, which finite element is used to analyze the cross section, the present study examines a simple polynomial based sectional analysis. The three-dimensional (3D) elasticity problem of the composite beam is decomposed into a two-dimensional (2D) cross section analysis and a one-dimensional (1D) beam analysis. In 2D cross-sectional problem in-plane and out-of-plane warpings are calculated by minimizing the energy functional with respect to warping functions. Simple polynomial series are employed to derive the necessary warping functions analytically. Cross sectional properties are determined as a fully populated 4×4 stiffness matrix. Based on a 1D beam potential and kinetic energy, natural frequencies of rotating and non-rotating beams are obtained. The presented approach is applied to isotropic, laminated composite beams as well as thin-walled composite box beams. The effects of fiber angle and rotational speed are investigated for flap and lag bending vibration. The accuracy of the results is validated in comparison with other theories, 3D FEM and experimental data. This procedure, which gives accurate predictions, eliminates the costly use of 3D finite element analysis.

1. Introduction

Composite materials are widely used in advanced engineering structures such as helicopter and wind turbine blades. The main benefit of using composite material in such structures is its high strength to weight ratio. Analysis of these structures due to directional behavior of composite materials and complex loading become completely sophisticated. On the other hand, a large amount of these composite structures is beam-like and can be modeled as a composite beam. Therefore, it is possible to take the advantage of small dimensional parameters of beam and have a lower dimensional structural model [1]. A desired theory of the composite beam should be applicable to beams with arbitrary cross-section geometry and consider all structural couplings between specified deformations. Moreover, it should include the influence of non-classical effects such as in-plane and out-of-plane warping, restrained warping and transverse shear effects. The in-plane and out-of-plane warping are cross-sectional displacements, which affect the accuracy of composite beam theory. The restrained warping is due to effect of restraining free warping displacement with the assumption of non-uniform torsion, across the beam length. The influence of restrained warping should be considered for open section beams [2]. Warping restraint effects are investigated for thin-walled composite beams in [3]. Generally, two types of cross-sectional warping are considered, namely, primary and secondary warping. Primary warping refers to warping displacement along the midline of the wall and secondary refers to warping of cross section points, off the midline contour [4]. Commonly, six degrees of freedom are considered for a beam model. Degrees of freedom are related to six fundamental deformations, which are extension, shear, twist, and bending in two orthogonal directions. In isotropic beam models, there would be six stiffnesses related to these deformations. Furthermore, by considering restrained warping, seven stiffnesses should be calculated. According to directional behavior of composite material, there would be material coupling between all mentioned degrees of freedom in a composite beam.

Popular composite beam theories can be divided into two groups. The first group use simplifying assumptions about displacement or stress field. The models are presented analytically but have restrictions for cross section geometry. Such theories use classical plate and shell assumptions and are commonly used for thin-walled beams. Chandra and Chopra [5] presented an experimental and theoretical vibration analysis of rotating thin-walled composite beams in 1992. The Galerkin method was used to analyze free vibration of composite box beams with bending-twist

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and extension-twist coupling. Kim and White [6] proposed a closed-section composite beam model in 1997. Non-classical effects such as primary and secondary torsional warping, transverse shear effects, of the cross-section and the beam walls were considered. An accurate analytical model was presented by Jung et al. [7] in 2002. In their paper, cross-sectional properties are computed as a 7×7 stiffness matrix, which can be implemented for both open and closed-section composite beams. Vo et. al. [8,9] presented a general analytical model for free vibration of thin-walled composite beams with arbitrary lay-up. The model considered all the structural coupling produced by material anisotropy. Sina and Haddadpour [10] in 2014 studied the axial-torsional vibrations of rotating pretwisted thin-walled composite beams. The influence of the primary and secondary warping was investigated.

The second group has focused on numerical modeling of the composite beam cross section with arbitrary geometry. Warping displacements are calculated numerically in order to reduce a 3D beam problem to a simple 1D beam model, with preserved non-classical effects. Working on composite beams with arbitrary cross section geometry started in the early 1980. Giavotto et al. [11,12] proposed a finite element based cross-sectional analysis of a composite beam. The cross-sectional properties of a beam with arbitrary cross section geometry and material anisotropy were derived in the form of a 6×6 stiffness matrix corresponding to the assumed six degrees of freedom. This work has led to the computer code HANBA2. Numerous well-known works were presented by Hodges and his colleagues [13–16]. The method is asymptotically correct finite element based sectional analysis. The associated computer program VABS is widely used in different structural field studies. The theory uses geometrically exact relations [17] and a simple mathematical method, VAM (Variational asymptotic method). The VAM is used to decompose a 3D elasticity problem into a 2D analysis of cross section and a 1D analysis across the beam length. Kim et al. [18] proposed a finite element based analysis for anisotropic beams with arbitrary-shaped cross section, which is named FAMBA. Formal asymptotic expansion method is used to obtain a set of 2D and 1D equations from 3D equilibrium equations. Sapountzakis et al. [19,20] developed a boundary element method based theory for composite beams with arbitrary cross section, considering warping and shear deformation effects. Carrera et al. [21] proposed a cross sectional beam analysis in which Carrera Unified Formulation (CUF) is used to develop a higher-order beam theory. Chakravarty [22] reviewed the modeling of composite beam cross sections in 2011. It is said that finite element based variational asymptotic beam sectional analysis (VABS) and boundary element method (BEM) are widely used for the analysis of composite beam cross sections.

In this paper, 3D beam elasticity is employed based on 3D beam kinematics of Ref [17]. The 3D beam problem is reduced to a 2D cross-sectional analysis by neglecting terms related to variation of warpings respect to beam reference line in the 3D strain field. In the cross-sectional analysis, in-plane and out-of-plane warping functions and stiffness matrix of cross section are derived. 2D analysis of cross section is accomplished by Rayleigh-Ritz method using simple polynomial functions. Therefore, the generated cross-sectional stiffness matrix contains 3D displacement effects and all considered structural couplings of cross-sectional degrees of freedom. In contrast to most of the previous work which employed FEM, using Rayleigh-Ritz in this article presents an analytical procedure in cross-sectional analysis and eliminates non-straightforward procedure of mesh generation on section. Using stiffness matrix of the cross section, stiffness and mass matrix of beam element is derived with the aid of a 1D strain and kinetic energy. Natural frequencies for rotating and non-rotating cantilever isotropic, laminated and composite box beams are calculated. The rotating beam element accounts for both centrifugal stiffening and softening matrices.

2. 3D beam model

The strain energy contains all geometry and material features of a structure. In order to extract a 1D beam problem without losing the 3D effects of a 3D elasticity model, introducing appropriate kinematics of deformation is vital. The aim is to find a strain energy containing warping displacements (3D deformation) and classical strains (1D deformation). Calculating the warping functions in terms of classical strains, leads to transforming the 3D beam problem into a 1D beam model. This procedure extremely simplifies the main problem while keeps 3D deformation effects in the beam model. The strain field of a beam contains kinematics of deformation and should consist of both warping and classical strain. 1D classical beam strain γ and the matrix of warping functions ω are considered as below:

$$\boldsymbol{\gamma} = \begin{bmatrix}\gamma_1 & \kappa_1 & \kappa_2 & \kappa_3\end{bmatrix}^{\mathrm{T}}$$
$$\boldsymbol{\omega} = \begin{bmatrix}\omega_1 & \omega_2 & \omega_3\end{bmatrix}^{\mathrm{T}} \tag{1}$$

Where γ_1 is extensional, κ_1 is twist and κ_2 and κ_3 are classical bending resultant strains related to the beam reference axis. Also ω_1 is out-of-plane and ω_2 and ω_3 are in-plane warping displacements. The indexes of 1D strains and warpings are according to direction of reference beam axes in Fig. 1. For example, ω_1 is displacement function of cross section in the x_1 direction and so on.

A nonlinear kinematics of a beam is presented in [17] by introducing the concept of decomposition of the rotation tensor. According to this kinematics of beam deformation, the 3D strain ε is given as follows [23]:

$$\boldsymbol{\epsilon} = G_\gamma \boldsymbol{\gamma} + G_h \boldsymbol{\omega} + G_l \boldsymbol{\omega}' \tag{2}$$

In Eq. (2) $\boldsymbol{\epsilon} = \begin{bmatrix}\epsilon_{11} & 2\epsilon_{12} & 2\epsilon_{13} & \epsilon_{22} & 2\epsilon_{23} & \epsilon_{33}\end{bmatrix}^{\mathrm{T}}$, $()'$ is the partial derivative with respect to x_1 and:

$$G_\epsilon = \begin{bmatrix} 1 & 0 & x_3 & -x_2 \\ 0 & -x_3 & 0 & 0 \\ 0 & x_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ G_h = \begin{bmatrix} 0 & 0 & 0 \\ \frac{\partial}{\partial x_2} & 0 & 0 \\ \frac{\partial}{\partial x_3} & 0 & 0 \\ 0 & \frac{\partial}{\partial x_2} & 0 \\ 0 & \frac{\partial}{\partial x_3} & \frac{\partial}{\partial x_2} \\ 0 & 0 & \frac{\partial}{\partial x_3} \end{bmatrix},\ G_l = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3}$$

Finding 3D strain field in the form of Eq. (2) is significant since it is linear respect to both warping and classical strain, which helps easily reduction of problem dimensions.

The 3D strain energy of a beam is then described by Eq. (4):

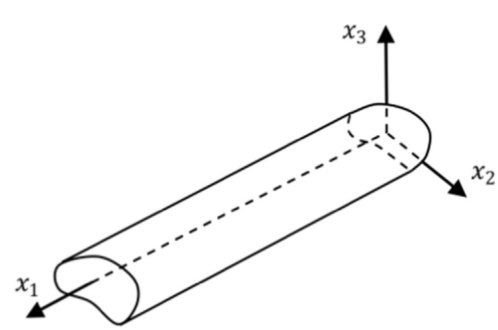

Fig. 1. Beam reference axes.