Author's Accepted Manuscript

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PII: S0020-7403(16)30098-4
DOI: http://dx.doi.org/10.1016/j.ijmecsci.2016.06.011
Reference: MS3323

To appear in: *International Journal of Mechanical Sciences*

Received date: 18 January 2016
Revised date: 16 May 2016
Accepted date: 16 June 2016

Cite this article as: Li Li, Yujin Hu and Xiaobai Li, Longitudinal vibration o size-dependent rods via nonlocal strain gradient theory, *International Journal c Mechanical Sciences,* http://dx.doi.org/10.1016/j.ijmecsci.2016.06.011

Longitudinal vibration of size-dependent rods via nonlocal strain gradient theory

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Abstract

The longitudinal vibration analysis of small-scaled rods is studied in the framework of the nonlocal strain gradient theory. The equations of motion and boundary conditions for the vibration analysis of small-scaled rods are derived by employing the Hamilton principle. The model contains a nonlocal parameter considering the significance of nonlocal elastic stress field and a material length scale parameter considering the significance of strain gradient stress field. The analytical solutions of predicting the natural frequencies and mode shapes of the rods with some specified boundary conditions are derived. A finite element method is developed and can be used to calculate the vibration problem by arbitrarily applying classical and non-classical boundary conditions. It is shown that the nonlocal strain gradient rod model exerts a stiffness-softening effect when the nonlocal parameter is larger than the material length scale parameter, and exerts a stiffness-hardening effect when the nonlocal parameter is smaller than the material length scale parameter. The higher-order frequencies are more sensitive to the non-classical boundary conditions in comparison with the lower-order frequencies, and the type of non-classical boundary conditions has a little effect on mode shapes.

Keywords:
Size-dependent rod, Vibration, Nonlocal strain gradient theory, Small-scaled effect

1. Introduction

Free vibration is a oscillation phenomenon occurring about an equilibrium point when a structural system is set in motion with some initial disturbance. The vibration analysis of a rod is necessary in many areas, such as mechanical, civil, aerospace and nano/micro engineering. The rapidly development of micro/nanotechnologies (including nanoelectromechanical (NEMS) and microelectromechanical systems (MEMS)) makes small-scaled rod possible to be used in potential designs and applications of nano/micro-scaled systems. Thus, the studies of size-dependent effect on the vibration analysis of size-dependent rods are always of fundamental significance and therefore have been drawing huge attention.

In general, experimental approaches, molecular dynamics (MD) simulations and continuum mechanics methods are often used for studying the mechanical properties of nano/micro-scaled structural systems. Experimental observation and MD simulations have shown significant small-scaled effects on the mechanical properties of nano/mirco-scaled systems. There is, however, often some difficult in MD simulations and experimental studies (often the MD simulations are complicated and time consuming and micro/nano-scaled controlled experiments are difficult to implement). To this end, size-dependent continuum mechanics models are usually used to predict the mechanical characteristics of micro/nano-scaled structural systems, including nonlocal continuum theories, modified couple stress theory (or modified strain gradient theory) and nonlocal strain gradient theory.

The nonlocal elasticity theory [1] hypothesized that the stress at a reference point should be dependent not only on the strains at the reference point but also on the strains at all other points in the domain. It has been shown by many studies that the nonlocal effect potentially plays a important role in studying the size-dependent effect on the vibration of rods [2–5], beams [6–13] and plates [14–17]. Most works studied these size-dependent effects based on the nonlocal model in differential form. It is recently reported by Fernández-Sáez et al. [18] that the nonlocal model in differential form is not generally equivalent to the nonlocal model in integral form, and the integral form need to be used for beam-type structures under some specified cases. The size-dependent mechanical behaviors of a bar in tension were also found to be inconsistent between the nonlocal model in differential and integral forms [19]. In general, the stiffness-softening effects were been observed in these nonlocal elasticity models.

The gradient elasticity theory [20] hypothesized that additional strain gradient terms should be considered by utilizing the assumption that small-scaled materials must be considered as atoms with higher-order deformation mechanism rather than just modeled as collections of points. The modified gradient elasticity theory (modified couple stress theory) [21] states that strain energy density must be considered as a function of both strain tensor conjugated with stress tensor and curvature tensor conjugated with couple stress tensor. It has been shown by many studies that the strain gradient effect also plays a important role in studying the size-dependent effect on the vibration

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