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Time-harmonic boundary value problem of coupled thermoelasticity and related integral equations method

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ABSTRACT

In this paper the regularities and criteria for the use of the linear theory of coupled thermoelasticity are determined. In this paper we considered axisymmetric problem of coupled thermoelasticity. For which a solution is obtained in closed form. Also, using the method of singular integral equations, boundary value problems for two-dimensional medium with curvilinear boundaries are solved. Numerical results are also presented.

1. Introduction

Recent studies in the field of thermoelasticity have shown that neglecting of the coupled summand in the system of thermoelasticity equations may be unreasonable. This is due to the fact that parameters of expedient consideration of the coupled thermoelasticity model may be implemented in modern electronic devices, microcircuits, micro-mechanical systems. The development of modern technology has provided practical importance and has drawn attention to the solving of coupled thermoelasticity problems.

It was shown that deformations didn't influence temperature distribution in most of the studies, so fields coupling can be neglected. Moreover, this fact is confirmed by engineering experiments and calculations while operating constructions parts with sizes in the range of 1 mm–1 m. Coupling effect is not significant in the unbounded thermoelastic medium during the dynamic processes in the sound frequency range, but it is significant at frequencies of $\omega^* = c(\lambda + 2\mu)/\lambda_T$ (c – thermal capacity, λ, μ – Lame's constants, λ_T – heat conduction coefficient) as it was shown in the earlier studies [11–13] devoted to the development of the coupled thermoelasticity theory. ω^* is approximately $10^{10} \sim 10^{11}$ c^{-1} for metals which corresponds to ultrasonic frequencies in the GHz range. On the other hand, ultrasonic vibrations are arisen in crystalline solids under the action of heat conductivity and mechanical energy which is dissipated because of defects [6]. Eringen [14] has noticed in his monograph that coupling of thermal and mechanical fields becomes significant at high-frequency and impulse actions while constructing the microelectronic devices.

Implementation of the above mentioned specific conditions in the technological development and manufacture of memory chips, microprocessors, micro- and nano-electromechanical systems stimulated the investigations of coupled thermoelasticity at the beginning of the XXI century. The heat conduction equation, which is the part of the coupled system of thermoelasticity, has been considered in detail in Ref. [23]. With the help of thermodynamic laws and different approaches to linearization, the author has found a criterion of applying of the coupled thermoelasticity theory: the term containing a derivative of deformation may not be included in the heat conduction equation if $\omega_0 t_0 \gg 1$, where t_0 – time scale of heat transfer process, ω_0 – frequency scale of the mechanical vibrations. The paper [7] deals with the initial value problems of the coupled thermoelasticity and it is mentioned that the coupling effect should be considered only in the range of high frequency elastic waves (the ultrasonic range). This range is implicitly linked to the size of the solid under consideration. Based on the approximate formula for the first normal frequency of the symmetrical elastic layer vibrations we have received an estimation for the typical range of elements to be studied as a part of the coupled theory: $r \approx 4, 44 \times 10^{-10}\sqrt{\mu/\rho}$. Table 1 shows the scale calculation

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Table 1
Dimension scale calculation for some materials within the coupled model of linear thermoelasticity.

Physical constants	Glass	Aluminum	Polycarbonate
Density ρ, kg/m^3	2230	2699	1200
Shear modulus μ, GPa	26.25	25	2.5
r, m	$\sim 10^{-9}$	$\sim 10^{-7}$	$\sim 10^{-7}$

for some materials.

Obviously, these scales are implemented in modern technological industry. For example, the dimension of modern circuits does not exceed 0.1 mm, and typical micro-mechanical ultrasonic sensors have the diameter of about 10 μm [25].

The works studying the thermoelastic vibrations of thin plates in micro-and nano-electromechanical systems (MEMS and NEMS) appeared during the last decade [10,18,20–22,24].

For example, the effects of thermoelastic damping in bending vibrations circular plates - circular micro-scale resonator are investigated in Ref. [24]. In numerical calculations, we took the resonators operating parameters that are used for infra-red ranging and detection and heat release [10]. Micro-scale circular resonators have thickness of 1–10 μm, the diameter of about 102–103 μm, and each one is operated at frequencies of 100–1000 MHz. Taking into account these parameters, the equation of coupled thermoelasticity for thin plates was solved in this paper. The article presents the comparison of the obtained results with the experimental data and other known works. Due to Euler-Bernoulli theory, J. Sharma and D. Grover received interrelated equations for transverse vibrations of thin beams with holes [22] and investigated the effect of thermal coupling and mechanical fields, holes location effect and beam size of energy dissipation which is caused by thermoelastic vibrations of resonators. A simplified analytical model of thermoelastic actuation of the micro-mechanical ultrasonic sensor is proposed in Ref. [18].

This work is devoted to investigation of thermoelastic harmonic vibrations of isotropic cylindrical solids with holes and plates with curved boundary under given boundary conditions on their surfaces. The problems are discussed in the two-dimensional setting.

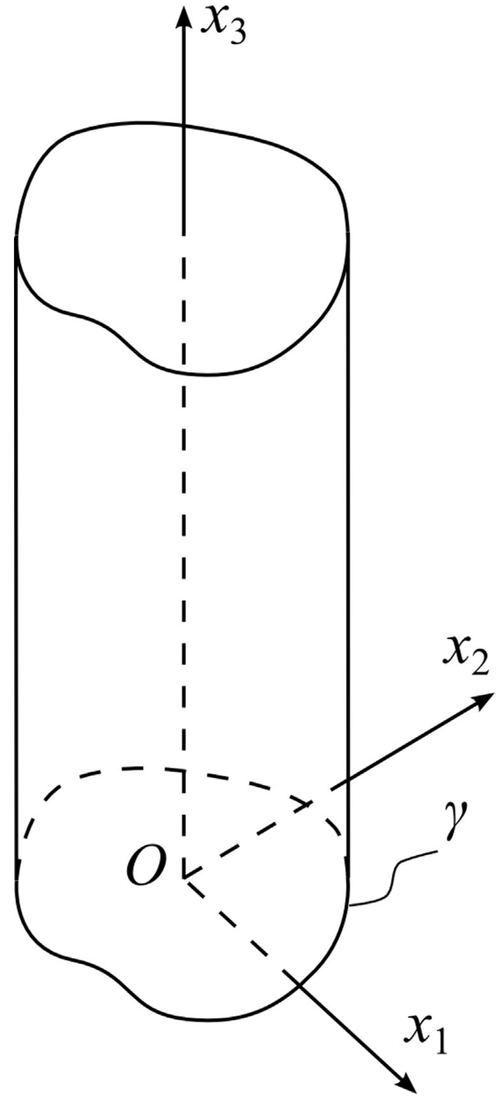

Fig. 1. 2d-medium under consideration.

2. Setting of the problem

Let us consider a solid limited by the cylindrical surface with parallel generating line to the axis Ox_3 in Cartesian coordinate system $Ox_1x_2x_3$. The plane x_1x_2, respectively, coincides with the plane of the cross-section surface (Fig. 1).

Let us assume that:

a) efforts are concentrated on the lateral surfaces and volumetric forces act in the planes which are perpendicular to the generating line, and do not vary in length;
b) deformations and temperature changes in solid are small;
c) surface and volumetric force are perpendicular to the generating line and do not depend on the coordinate x_3. Under these conditions, we have the problem of planar deformation in the plane x_1Ox_2 (domain D is limited by cam contour γ). On the cylindrical surface the boundary conditions are set in terms of stresses and conditions of heat transfer by Newton's law.

The complete system of dynamic thermoelasticity equations for two-dimensional isotropic medium is as follows [15,16]:

coupled system of equations for two-dimensional isotropic medium (planar deformation)

$$\nabla^2 u_1 + \sigma\partial_1 e - \frac{\beta}{\mu}\partial_1 T - \frac{\rho}{\mu}\partial_t^2 u_1 = -\frac{1}{\mu}f_1(x,t),$$
$$\nabla^2 u_2 + \sigma\partial_2 e - \frac{\beta}{\mu}\partial_2 T - \frac{\rho}{\mu}\partial_t^2 u_2 = -\frac{1}{\mu}f_2(x,t),$$
$$\nabla^2 T - \frac{1}{a^2}\partial_t T - \eta\partial_t e = -\frac{1}{\lambda_T}w(x,t), \tag{1.1}$$

$$e = \operatorname{div}\vec{u},\ \eta = \beta T_0/\lambda_T,\ \beta = \alpha_T(3\lambda + 2\mu),\ \sigma = (\lambda+\mu)/\mu,$$
$$a^2 = \lambda_T/c\rho,\ \partial_j = \partial/\partial x_j,$$
$$\partial_t = \partial/\partial t,\ x = (x_1, x_2),\ \nabla^2 = \partial_1^2 + \partial_2^2$$

Duhamel-Neumann relation

$$\sigma_{ij} = \mu\left(\partial_j u_i + \partial_i u_j\right) + \left[\lambda e - \beta T\right]\delta_{ij} \tag{1.2}$$

boundary conditions on the contour of cross-section cylindrical surface.

$$\sigma_{ij}n_j\big|_\gamma = y_j(x,t),\ t>0$$
$$\left[\lambda_T\frac{\partial T}{\partial n} + \alpha_\gamma\left(T - T_\gamma\right)\right]_\gamma = 0, \tag{1.3}$$