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Exact determination of the global tip deflection of both close-coiled and open-coiled cylindrical helical compression springs having arbitrary doubly-symmetric cross-sections

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ABSTRACT

A general formulation of the static deflection under an axial force is required for accurate static, buckling and dynamic analyses. Even today, however, the helical spring formulas derived in 1960s still continue to be used in the spring design. So designers maintain to design helical springs with limited options in making a change in both the helix pitch angles and cross-section types. In this study, in order to carry out such formulation and get closed-form solutions for the vertical tip deflection, Castigliano's first theorem is directly employed to the linear elastic problem of cylindrical helical springs with large pitch angles. Derivation takes into account for the whole effect of the stress resultants such as axial and shearing forces, bending and torsional moments on the deformations. Cylindrical helical springs having doubly symmetric cross-sections such as a solid/hollow circle, a square, a horizontal/vertical rectangle, and a horizontal/vertical ellipse made of isotropic and homogeneous linear elastic materials are all handled in this work. For each shape of cross-section considered in the study, a closed form global formula in a compact form is offered for users with the common notations and common design parameters as currently used. These formulas may be directly used without hesitation for both closed-coiled (CC), $\alpha \le 10^{\circ}$, and open-coiled (OC), $\alpha \ge 10^{\circ}$, cylindrical helical springs. That is one may use those formulas without the need for any extra information than he already has and without involving any design chart and correction factor. Some of formulas derived in this study are compared to the commonly used formulas in the available literature. It is verified that those formulas may be obtained readily from the present formulas by considering their certain assumptions. Benefits of this study related to previous ones are also discussed. Present global formulas are also verified with available recent experiments and finite element solutions. The groundwork for the data to be used directly in Castigliano's first theorem was obtained by using differential geometry of a helical structure from a general spatial rod, and governing equations derived by using equilibrium equations, constitutive equations and geometrical compatibility relations in vector forms. As a result, the author expects that a designer is to be free to design more accurate springs by using the global analytical formulas presented in this study.

1. Introduction

A helical spring is a topic of interest in mechanical engineering design. That may be encountered in practice too many problems may be solved by using cylindrical helical springs which are also referred to as coil springs. Cylindrical helical springs cover a wide-range application area and hold an intense know-how about their manufacturing. So they are considered separately from non-cylindrical helical springs in this study.

Wahl [1] summarized spring research up to the year 1963. Wahl's work [1] is assumed to be a "bible" of the spring design by many spring makers. This authoritative work on spring design emphasizes the widely used isotropic helical compression and extension springs. Even today the helical spring formulas derived by Wahl [1] in 1960s still continue to be used in the design. These formulas are used in particular conditions/sections and do not consider the whole effect of the stress resultants such as axial and shearing forces, bending and torsional moments on the deformations at the same instant. So they cannot referred to as global. They are generally valid for small helix pitch angles, $\alpha \le 10^{o}$, relatively large spring indexes, $C=D/d \ge 10$, circular/square cross-sections and right cylinders.

In general, the effects of the axial force and bending moment are not considered in the spring formulation. Shearing force effects are generally considered by employing some correction factors in

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existing formulas. For instance, Timoshenko [2] offered such a formula by taking into consideration the whole effects of the shearing force, bending and torsional moments except the axial force for springs with circular sections and large pitch angles. By using a “thin slice” method which allows to reduce the three variable problem in elasticity in two variables, Ancker and Goodier [3–5] have obtained more accurate solutions based on elastic theory for cylindrical helical springs with circular sections. They took combined effects of pitch angle and coil curvature into account and assumed that the wire diameter of the helix does not change appreciably during deflection. In Ancker and Goodier's [1,3–5] formulas, Wahl's [1] non-corrected formula is again multiplied by a factor which depends on the helix index and helix pitch angle. There are two different forms frequently used of those formulas in the literature. In general, Ancker and Goodier's [1,3–5] formulas are used to improve more accurate finite elements for general helical structures.

The existing formulas for general cross section shapes other than circular are too limited. For instance the existing formula given for rectangular shape by Wahl [1] cannot be used directly without resorting to the design charts to determine numerical values of some constants which emerge in the formulas as seen in Fig. 1. That is in the classical approach in order to the design of springs having different cross section shapes rather than solid circle, accessible proposed formulas should require simultaneously use of some additional charts and tables. This kind of design process may also end in chart-reading errors.

As stated above, a spring design is often involved elementary formulas together with tables and charts containing certain preselected specifications and objectives. Spring design takes considerable time due to the trial and error method. Along with the development in computers, some commercial software programs based on existing analytical and empirical equations together with some design charts are suggested for the designers.

A true buckling formulation even in a numerical analysis also absolutely expects global deflection formulas which precisely define a pre-buckled deformed configuration of the spring at the buckling instant [6–9]. Determination of the true geometrical properties immediately before the buckling instant of the helix may be done numerically with consuming substantial effort and time. To simplify the buckling problem considerably and to save an amount of computing time, for large helix pitch angles, in general this is carried out by using the existing analytical formulas which

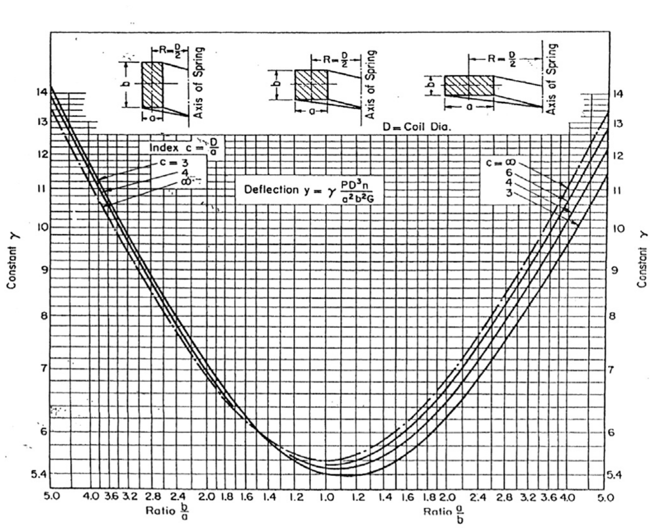

Fig. 1. Variation of the constant γ with respect to the spring index and height/base ratios of rectangular cross sections [1].

are included into iterations to yield an initial helix angle corresponding to the critical buckling load [10–17]. That is the same problem is to be solved more than one times until getting a satisfactory equality of the initial helix angle and buckled helix angle. In recent times, Patil et al. [18–21] gave a considerable attendance on the development closed-form buckling equations for both cylindrical and non-cylindrical helical springs having circular sections. They also tried to verify their analytical results by their experiments performed for close-coiled springs. In one of their studies Patil et al. [20] considered the effect of shearing force that was previously ignored for a conical spring in their formulation to eliminate the differences between theory and experiments.

Although dynamic behavior of isotropic and homogeneous cylindrical helical springs has been widely studied by now, there are a little numerical [22–32] and analytical/experimental [32–37] works on static response of cylindrical helical springs. From those Dym's [36] exceptional study is also worthy of mention. For the first time in the literature, Dym [36] derived the spring rate of a coiled cylindrical extensional helical spring with solid circular wire under an axial force and an axially directed torque by a consistent application of Castigliano's second theorem, and showed that the coupling between the two loads may not always be neglected. Dym's [36] exceptional study presenting the common notion about the effects of each stress resultants on the deformation of the spring, unfortunately, was performed for only cylindrical helical springs with solid circular sections without presenting any formula for practical use. Haktanır [35] also worked out the static behavior of isotropic cylindrical and non-cylindrical helical springs (barrel, hyperboloidal and conical types) subjected to an axial static force. He presented analytical formulas for the vertical tip deflection of those springs by considering the whole effects of the stress resultants, large helix pitch angles and different types of cross sections with the help of the Castigliano's first theorem.

In the realm of civil engineering and biomechanical applications, a helical geometry is also used in the form of other helical construction types such as helical cables (strands) and helical carbon nanotubes. Although this type of helical structures are out of scope of the present study, it may be useful to mention about few references in connection with them [37–41]. By using the homogenization theory in a twisted coordinate system, Frikha et al. [40] reduced the model to a 2D one, i.e. a cross-section model to consider helical symmetry more efficiently in helical structures. The method developed in [40] was restricted to multi-wire helical structures composed of a stack of helical wires wrapped with the same twisting rate around a straight axis. This approach was validated through comparison with analytical solution suggested by Ancker and Goodier [3] for helical single wire structures and 3D detailed finite element solution for seven-wire strands. Nikolas and Gerald [41] employed existing analytical theory for helical cables and incorporated the radial deformation in the theory as an additional kinematic degree of freedom. They verified their results with the use of a finite element model over a wide range of helix angular configurations.

The differences between elementary formulas and experiments still continue to exist. In general, for each fresh situation encountered, the researches try to fit the new problem to the existing formulas by offering additional coefficients as correction factors. This kind of studies together with the existing ones also lead to confusion in the minds of especially beginners.

Paredes [37] developed experiments to test the accuracy of the common formulae given by Wahl [1] that define spring's static behavior. Paredes [37] tested cylindrical compression springs with a constant pitch. Paredes [37] showed that the common formulae offered by Wahl [1] gave quite accurate results when at least five free coils were considered for springs with closed and ground ends but results could be of poor accuracy for large indexes and few free