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Post-buckling behaviour of shear deformable functionally graded curved shell panel under edge compression

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In this article, the post-buckling behaviour of functionally graded curved shell panels of different shell geometries (spherical, elliptical, cylindrical and hyperbolic) are investigated under the uniaxial and the biaxial edge compression. The inhomogeneity of the functionally graded material along the thickness direction is achieved using power-law distribution through Voigt's micromechanical model to obtain the effective material properties. The kinematic model is developed with the help of higher-order shear deformation theory in conjunction with Green-Lagrange geometrical nonlinear strains. The governing equation of the axially loaded functionally graded curved panel is derived using the minimum total potential energy principle. The nonlinear finite element steps have been employed to discretise the shell panel domain and solved with the help of Picard's iterative method to obtain the desired load parameter. Further, the effects of different parameters such as the amplitude ratios, the power-law indices, the curvature ratios, the thickness ratios, and the support conditions on the buckling and post-buckling responses of the functionally graded curved panels are demonstrated through suitable numerical illustrations.

1. Introduction

Functionally graded materials (FGMs) are now being adopted for the structural component in aerospace industry due to their tailor-made characteristics along the thickness direction. The structures subjected to in-plane loading are prone to buckle. However, buckling does not confirm the ultimate failure of any structure and can carry an extra amount of load after buckling is called post-buckling. In this regard, the stability of functionally graded (FG) structures has been examined and reported by many previous works. Different solution techniques and shell/plate theories have been adopted to execute the necessary investigation and few of the recent research contributions are discussed in the following line to point out the knowledge gap.

Sofiyev [1–3] used Galerkin method to study the buckling of FG cylindrical and truncated conical shells under internal and external pressure. Duc and Tung [4,5] employed the Galerkin method to analyse the post-buckling of FG cylindrical panels subjected to axial compression by considering the initial geometrical imperfection. They employed the classical laminated plate theory (CLPT) and first-order shear deformation theory (FSDT) mid-plane kinematics and von-Karman–Donnell type geometrical nonlinearity. Yang et al. [6,7] investigated the post-buckling behaviour of FG cylindrical shell panel under the thermomechanical load using the semi-analytical differential quadrature-Galerkin method. The authors used the CLPT and the higher-order shear deformation theory (HSDT) mid-plane kinematics in conjunction with von-Karman–Donnell-type nonlinear strain. Duc and Quan [8] examined analytically the thermomechanical buckling and post-buckling behaviour of the thick doubly-curved FG shallow panels resting on elastic foundations using the Galerkin method and CLPT mid-plane kinematics. Wu et al. [9] examined the post-buckling responses of FG flat panel using the finite double Chebyshev polynomial approach and the FSDT based von-Karman nonlinear strain. Zhao et al. [10,11] examined the buckling behaviour of FG flat panel subjected to in-plane mechanical and thermal load using the element-free kp-Ritz method and the FSDT mid-plane kinematics. Shahsiah et al. [12] utilised the FSDT based Sanders kinematics to obtain the stability equations for simply supported FG spherical deep shells. Liew et al. [13] examined the post-buckling behaviour of FG cylindrical shell panels under the edge compression and thermal loading using the FSDT based von-Karman strain field and the element-free kp-Ritz method. Woo et al. [14] used the HSDT based von-Karman large deflection theory to obtain the post-buckling responses of FG flat/cylindrical panels under in-plane loadings. Shen [15–21] employed the CLPT/HSDT mid-plane kinematics based von-Karman-Donnell-type strain field to examine the stability of FG flat/cylindrical panels

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subjected to various loading conditions. Huang and Han [22–27] study the buckling/post-buckling responses of temperature-dependent cylindrical FG shells using the Donnell shell theory. Darabi et al. [28] utilised the Galerkin approach for nonlinear stability of cylindrical FG shell subjected to periodic in-plane loading. Ebrahimi and Sepiani [29] examined the dynamic stability of FG cylindrical shells under the combined loading by including the transverse shear and rotary inertia effect. Tung [30] studied the nonlinear thermomechanical stability of FG circular plate and spherical panel using the Galerkin method and the FSDT kinematics. Thai and Choi [31] employed the simple refined theory to investigate the buckling behaviour of FG plates for various support conditions. Valizadeh et al. [32] utilised the NURBS based Bubnov-Galerkin iso-geometric finite element method (FEM) and FSDT displacement field to study the stability of FG flat panels under the thermal field. Bakora and Tounsi [33] used the Galerkin technique and the HSDT based von- Karman kinematics to study the post-buckling of FG flat panel subjected to various loading conditions. Meksi et al. [34] proposed a simple FSDT by including the neutral surface position and subsequently performed the static and dynamic study of FG plates. Bennoun et al. [35] developed a new five-variable refined plate theory to examine the free vibration of FG sandwich plates. Singh and Babu [36] examined the thermal buckling responses of laminated conical shell panel embedded with and without piezoelectric layer using the HSDT kinematics. Lal et al. [37] used the HSDT based von-Karman nonlinear kinematics to study the thermomechanical post-buckling of FG plate with and without cut-outs.

We note from the available literature that most of the studies reported on the buckling and/or post-buckling behaviour of the flat and single-curved shell panels under the uniaxial and the biaxial load. Based on the authors' knowledge, the post-buckling behaviour of the FG doubly-curved shell panel using the HSDT and Green-Lagrange kinematics has not yet been reported in open literature. Hence, in this article authors have attempted to examine the critical buckling and post-buckling load parameters of the FG curved shell panels of different geometries (spherical, elliptical, cylindrical and hyperbolic) under both the uniaxial and the biaxial edge compression except the conical shell panel. It is also important to mention that the present numerical model includes all the nonlinear higher-order terms to evaluate the geometry matrix and the nonlinear stiffness matrices to capture the geometrical distortion under the biaxial compression loading for the exact buckling and the post-buckling strength.

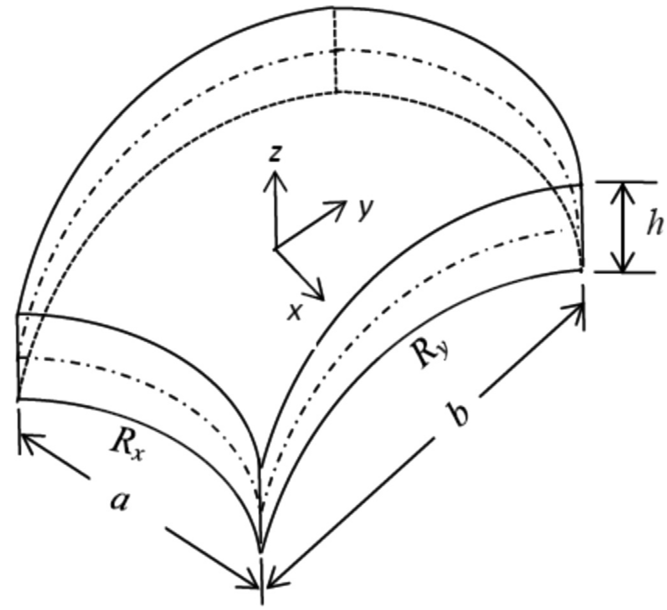

Fig. 1. Geometry and dimension of doubly-curved shell panel.

2. General mathematical formulation

In the present analysis, a generalised doubly-curved FG shallow shell panel of uniform thickness 'h' with principal radii R_x and R_y is considered in a rectangular base of sides a and b, and presented in Fig. 1. Different shell configurations such as spherical (R_x=R_y=R), elliptical (R_x=R; R_y=2 R), hyperbolic (R_x=R; R_y=-R) and cylindrical (R_x=R; R_y=∞) panels are be modelled from the present doubly-curved shell panel by varying the curvature radii.

2.1. Displacement field

The HSDT with nine degrees-of-freedom is used to model the displacement field of FG curved panel in the mid-plane (z=0) [38].

$$\left.\begin{aligned} u &= u_0 + z\theta_x + z^2 u_0^* + z^3\theta_x^* \\ v &= v_0 + z\theta_y + z^2 v_0^* + z^3\theta_y^* \\ w &= w_0 \end{aligned}\right\} \tag{1}$$

where, $(u,\ v,\ w)$ and $(u_0,\ v_0,\ w_0)$ are the global and mid-plane displacements in $(x,\ y,\ z)$ directions, respectively. $(\theta_y,\ \theta_x)$ and $\left(u_0^*,\ v_0^*,\ \theta_x^*,\ \theta_y^*\right)$ are the shear rotations and the higher-order terms.

2.2. Strain-displacement field

The strain-displacement field $\varepsilon = \left\{\varepsilon_{xx}\ \ \varepsilon_{yy}\ \ \gamma_{xy}\ \ \gamma_{xz}\ \ \gamma_{yz}\right\}^T$ of the FG curved shell panel is defined in the Green-Lagrange sense [38].

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \left(\dfrac{\partial u}{\partial x} + \dfrac{w}{R_x}\right) \\ \left(\dfrac{\partial v}{\partial y} + \dfrac{w}{R_y}\right) \\ \left(\dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} + \dfrac{2w}{R_{xy}}\right) \\ \left(\dfrac{\partial u}{\partial z} + \dfrac{\partial w}{\partial x} - \dfrac{u}{R_x}\right) \\ \left(\dfrac{\partial v}{\partial z} + \dfrac{\partial w}{\partial y} - \dfrac{v}{R_y}\right) \end{Bmatrix} + \begin{Bmatrix} \dfrac{1}{2}\left[\left(\dfrac{\partial u}{\partial x} + \dfrac{w}{R_x}\right)^2 + \left(\dfrac{\partial v}{\partial x} + \dfrac{w}{R_{xy}}\right)^2 + \left(\dfrac{\partial w}{\partial x} - \dfrac{u}{R_x}\right)^2\right] \\ \dfrac{1}{2}\left[\left(\dfrac{\partial u}{\partial y} + \dfrac{w}{R_{xy}}\right)^2 + \left(\dfrac{\partial v}{\partial y} + \dfrac{w}{R_y}\right)^2 + \left(\dfrac{\partial w}{\partial y} - \dfrac{v}{R_y}\right)^2\right] \\ \left[\left(\dfrac{\partial u}{\partial x} + \dfrac{w}{R_x}\right)\left(\dfrac{\partial u}{\partial y} + \dfrac{w}{R_{xy}}\right) + \left(\dfrac{\partial v}{\partial x} + \dfrac{w}{R_{xy}}\right)\left(\dfrac{\partial v}{\partial y} + \dfrac{w}{R_y}\right) + \left(\dfrac{\partial w}{\partial x} - \dfrac{u}{R_x}\right)\left(\dfrac{\partial w}{\partial y} - \dfrac{v}{R_y}\right)\right] \\ \left[\dfrac{\partial u}{\partial z}\left(\dfrac{\partial u}{\partial x} + \dfrac{w}{R_x}\right) + \dfrac{\partial v}{\partial z}\left(\dfrac{\partial v}{\partial x} + \dfrac{w}{R_{xy}}\right) + \dfrac{\partial w}{\partial z}\left(\dfrac{\partial w}{\partial x} - \dfrac{u}{R_x}\right)\right] \\ \left[\dfrac{\partial u}{\partial z}\left(\dfrac{\partial u}{\partial y} + \dfrac{w}{R_{xy}}\right) + \dfrac{\partial v}{\partial z}\left(\dfrac{\partial v}{\partial y} + \dfrac{w}{R_y}\right) + \dfrac{\partial w}{\partial z}\left(\dfrac{\partial w}{\partial y} - \dfrac{v}{R_y}\right)\right] \end{Bmatrix} \tag{2}$$

$$\{\varepsilon\} = \{\varepsilon_l\} + \{\varepsilon_{nl}\} \tag{3}$$

The linear $\{\varepsilon_l\}$ and the nonlinear $\{\varepsilon_{nl}\}$ strain tensors can be rewritten in terms of mid-plane strain vectors $(\{\bar{\varepsilon}_l\}, \{\bar{\varepsilon}_{nl}\})$ and thickness co-ordinate matrices $([T_l],\ [T_{nl}])$.