



# Vibration analysis of CNT-reinforced functionally graded composite cylindrical shells in thermal environments

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## ABSTRACT

Vibration characteristics of CNT-reinforced functionally graded composite closed cylindrical shells are studied. Thermal effect is taken into account. In the structural modeling, Reddy's high-order shear deformation theory is applied. Hamilton's principle and the assumed mode method are used to formulate the equation of motion of the CNT-reinforced functionally graded cylindrical shell. Vibration properties of the cylindrical shell are analyzed through the time- and frequency-domain methods. Influences of temperature change, CNTs distribution as well as CNTs volume fractions on the natural frequency of the CNT-reinforced cylindrical shell are investigated. Vibration responses of the cylindrical shell computed by the FSDT and TSDT are compared to verify the necessity of the high-order shear deformation theory in the vibration analysis for thick CNT-reinforced structures. The effects of CNTs volume and distribution on the free and forced vibration of the cylindrical shell are studied. The influences of thermal effect on the vibration responses of the CNT-reinforced cylindrical shell are also investigated.

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## 1. Introduction

In recent years, CNTs have attracted more and more attentions of researchers for their excellent mechanical properties. It is well known that addition of CNTs in a matrix can improve the thermal and physical behaviors significantly. Usually the CNTs in a polymeric matrix are functionally graded, which are known as the FG-CNT-reinforced composite materials. A large number of literatures have researched the buckling, vibration and bending behaviors of CNT-reinforced functionally graded composite structures [1]. Shen and Xiang [2] investigated the large amplitude vibration behavior of nanocomposite cylindrical shells reinforced by single-walled carbon nanotubes in thermal environments. Lin and Xiang investigated the linear [3] and nonlinear [4] free vibration characteristics FG-CNT beams based on the first- and third-order shear deformation theories. Mehri et al. [5] presented a research dealing with bifurcation and vibration responses of composite truncated conical shell with embedded single-walled CNTs under external pressure and axial compression. Ansari and Torabi [6] employed an efficient numerical method to study the buckling and vibration of axially-compressed functionally graded CNT-reinforced conical shells. Mirzaei and Kiani

[7] studied the linear thermal buckling of CNT-reinforced conical shells. The postbuckling behaviors of CNT-reinforced functionally graded cylindrical shells subjected to either axial compression or lateral pressure in thermal environments were studied by Shen [8]. Zhu et al. [9] presented the bending and free vibration analyses of CNT-reinforced composite plate using the finite element method. Using the novel element-free IMLS-Ritz method [10,11], Zhang et al. [12–16] studied the flexural strength, free vibration, buckling and nonlinear bending behaviors of FG-CNT-reinforced composite plates and cylindrical panels of various platforms with different boundary conditions. The first-order shear deformation theory (FSDT) was applied to the structural modeling. García-Macías et al. [17] provided numerical results for static and dynamic simulations of CNT-reinforced skew plates. Zhang et al. [18] and Lei et al. [19–21] investigated the static, dynamic and buckling properties of FG-CNT-reinforced composite plates, laminated plates and rotating cylindrical panels using the element-free kp-Ritz method. They also conducted free vibration analyses of FG-CNT-reinforced composite plates, considering the thermal effects [22]. Based on the state-space Levy solution, Zhang et al. [23] intelligently studied the vibration and buckling behaviors of high-order FG-CNT composite plates.

In mechanical and civil engineering, closed cylindrical shells are very common structures. Therefore it is necessary to study the dynamic properties of closed cylindrical shell. There are also a large number of literatures that have studied the vibration behaviors of

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cylindrical shells. Zhang et al. [24] presented the vibration analysis of cylindrical shells using wave propagation method. Xing et al. [25] given out the exact solutions for free vibration of circular cylindrical shells with classical boundary conditions based on the Donnell–Mushtari shell theory. Xie et al. [26] presented a novel and efficient solution for free vibrations of thin cylindrical shells subjected to various boundary conditions by using the Haar wavelet discretization method. Reddy and Liu [27] developed a higher-order shear deformation theory of elastic shells for shells laminated of orthotropic layers. Palazotto et al. [28] conducted an analytical study to determine the fundamental frequencies and critical buckling loads for laminated anisotropic circular cylindrical shell panels. Jin et al. [29] developed a unified analytical method based on the first-order shear deformation theory for the vibration analysis of moderately thick composite laminated cylindrical shells subjected to general boundary conditions and arbitrary intermediate ring supports. Lopatin and Morozov [30] studied the free vibrations of laminated composite cylindrical shell with clamped edges.

From the literature review, it is noted that a large number of literatures have studied the static, buckling and vibration behaviors of CNT-reinforced functionally graded composite plate. However, vibration properties and active control of high-order CNT-reinforced functionally graded composite cylindrical shell have not been fully studied. In our recently work, we have presented the free and forced vibration behaviors of CNT-reinforced functionally graded plate [31]. The active vibration control has also been carried out. In this paper, vibration characteristics for the CNT-reinforced functionally graded closed cylindrical shell will be further investigated. In the structural modeling, Reddy's high-order shear deformation theory will be used. Three types of CNT distribution will be considered. The equation of motion of the CNT-reinforced composite cylindrical shell will be formulated by the Hamilton's principle. The effects of CNTs volume fraction and distribution on the frequency and vibration responses of the cylindrical shell will be studied.

## 2. Effective material properties

Although the damping properties of CNT-reinforced structures can be improved due to the stick-slip frictional motion of CNTs [32], the micromechanical nanotube/resin interaction model is neglected in this investigation, and as most of the open literatures [2,6,8], we only focus our study on the macro mechanical characteristics of CNT-reinforced cylindrical shells. In this study, the extended rule of mixture is used to determine the effective material properties of the CNT-reinforced composites. Three types (FGX, FGO and UD) [2] of CNTs distributions for the CNT-reinforced functionally graded composite cylindrical shells are considered. The CNTs align in the axial direction of the cylindrical shell. In FGO, the mid-plane of the structure is CNT-rich. As for the FGX, both the top and bottom surfaces of the shell are CNT-rich. The volume fractions of the three distribution types are expressed as follows:

$$V_{CNT}(\zeta) = V^*, \quad (\text{UD}) \quad (1a)$$

$$V_{CNT}(\zeta) = 2\left(1 - \frac{2|\zeta|}{h}\right)V^*, \quad (\text{FGO}) \quad (1b)$$

$$V_{CNT}(\zeta) = \frac{4|\zeta|}{h}V^*, \quad (\text{FGX}) \quad (1c)$$

where  $V^* = \rho^m w_{CNT} / (\rho^m w_{CNT} + \rho^{CNT} w_{CNT} + \rho^{CNT})$ , in which  $w_{CNT}$  is the mass fraction of the CNTs, and  $\rho^{CNT}$  and  $\rho^m$  are densities of the CNTs and matrix, respectively. According to the extended rule of mixture,

the effective material properties of CNT-reinforced functionally graded composite cylindrical shell can be expressed as [33–35]:

$$\begin{aligned} \alpha_{11} &= \frac{V_{CNT}(\zeta)E_{11}^{CNT}\alpha_{11}^{CNT} + V_m(\zeta)E^m\alpha^m}{V_{CNT}(\zeta)E_{11}^{CNT} + V_m(\zeta)E^m}, \quad \alpha_{22} = (1 + \nu_{12}^{CNT})V_{CNT}(\zeta)\alpha_{22}^{CNT} \\ &\quad + (1 + \nu^m)V_m(\zeta)\alpha^m - \nu_{12}\alpha_{11}, \\ E_{11} &= \eta_1 V_{CNT}(\zeta)E_{11}^{CNT} + V_m(\zeta)E^m, \\ \frac{\eta_2}{E_{22}} &= \frac{V_{CNT}(\zeta)}{E_{22}^{CNT}} + \frac{V_m(\zeta)}{E^m}, \quad \frac{\eta_3}{G_{12}} = \frac{V_{CNT}(\zeta)}{G_{12}^{CNT}} + \frac{V_m(\zeta)}{G^m}, \\ \nu_{12} &= V^*\nu_{12}^{CNT} + V_m\nu^m, \quad \nu_{21} = \nu_{12}E_{22}/E_{11}, \\ \rho(\zeta) &= V_{CNT}(\zeta)\rho^{CNT} + V_m(\zeta)\rho^m, \end{aligned} \quad (2)$$

where  $E^{CNT}$ ,  $G^{CNT}$  and  $\alpha^{CNT}$  are the Young's moduli and thermal expansion coefficient of CNTs,  $E^m$ ,  $G^m$  and  $\alpha^m$  are the corresponding parameters of the isotropic matrix, and  $\nu^{CNT}$  and  $\nu^m$  are the Poisson's ratios of the CNTs and matrix. It should be pointed out that the thermal expansion effect that can affect the stiffness of the CNT-reinforced cylindrical shell are taken into account, the effective coefficients of which are given out in Eq. (2). Since the load transfer between the CNT and matrix is less than perfect, the CNT efficiency parameters  $\eta_1$ ,  $\eta_2$  and  $\eta_3$  are introduced to account for load transfer between the CNT and polymeric phases.  $V_{CNT}$  and  $V_m$  are the volume fractions of the CNT and matrix, and their sum must be equal to 1, that is  $V_{CNT} + V_m = 1$ .

## 3. Formulation of the equation of motion

The CNT-reinforced functionally graded composite closed cylindrical shell studied in this investigation is shown in Fig. 1. The radius, thickness and length of the cylindrical shell are  $R$ ,  $h$  and  $L$ , respectively. Based on the Reddy's third-order shear deformation theory, the displacement fields of the cylindrical shell have the following form:

$$\begin{aligned} u &= u_0 + \zeta\phi_1 - \frac{4}{3h^2}\zeta^3\left(\phi_1 + \frac{\partial w_0}{\partial x}\right), \quad v = \left(1 + \frac{\zeta}{R}\right)v_0 \\ &\quad + \zeta\phi_2 - \frac{4}{3h^2}\zeta^3\left(\phi_2 + \frac{1}{R}\frac{\partial w_0}{\partial \theta}\right), \quad w = w_0, \end{aligned} \quad (3)$$

where  $u$ ,  $v$  and  $w$  are the displacements at any point  $(x, \theta, \zeta)$  of the cylindrical shell,  $u_0$ ,  $v_0$  and  $w_0$  are the in-plane and transverse displacements in the neutral plane,  $\phi_1$  and  $\phi_2$  are the rotations of the transverse normal about the  $\theta$  and  $x$  axes, and  $\zeta$  is the transverse coordinate.

Substituting Eq. (3) into the strain–displacement relations [36,37], the following expressions can be obtained:

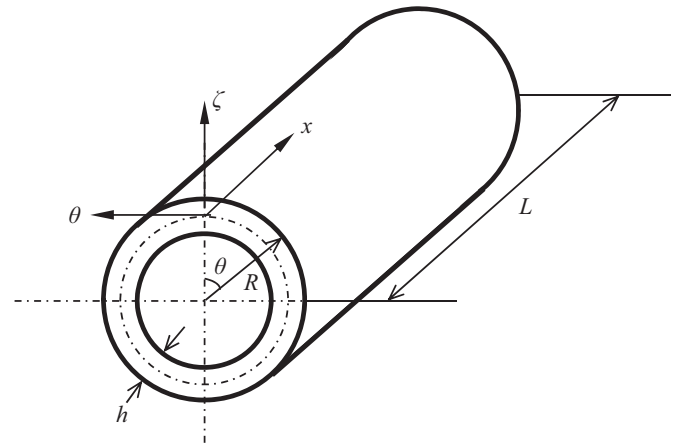


Fig. 1. Schematic diagram of the cylindrical shell.

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