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# Free in-plane vibration analysis of rotating rectangular orthotropic cantilever plates

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ABSTRACT

The aim of the present work is to provide results for in-plane vibrations of rotating orthotropic cantilever plates. To this end, the equations of motion include the effect of Coriolis are derived by Hamilton's principle and then solved using a Extended Kantorovich Method (EKM). The EKM results are validated by comparison with the results obtained via a Extended Galerkin Method and compared by available results in the literatures which show good agreement. The effects of the dimensionless parameters on the natural frequency are investigated and the resonant frequencies and Tune speeds are also presented and discussed. The observation of the results in different cases shows that increasing speed of rotation may vary the natural frequencies and the rate of these variations is dependent on the stiffness ratio. Incidentally, the parametric analysis on the in-plane mode shapes of rectangular orthotropic rotating plates is conducted.

## 1. Introduction

In several types of engineering structures, rotating blades appear as components, such as turbine blades and turbomachines. A better understanding of the dynamic behaviors of rotating systems is a key factor for their robust and reliable design of such type of structures and so dynamic properties should be identified properly. It is well known that modal characteristics of these structures due to the spinning motion often change significantly compared to those of non-rotating structures.

Rotating blades are commonly modeled as rotating cantilever beams that are the one-dimensional type model. Many authors have already studied on this subject, for example Piovan and Sampaio [1] studied the dynamic behavior of rotating beams made of functionally graded materials. The influence of the graded properties and geometric stiffening of the rotating beam by using finite element method are figured out. Natural frequency of very slender rotating Euler beam at high angular velocity was investigated by Huang et al. [2]. Li et al. [3] developed a dynamic model of a rotating hub–functionally graded material beam system to study its free vibration characteristics. Dynamic equations were derived using the method of assumed modes and Lagrange's equations of the second kind.

Although lots of rotating structures can be handled as beams, they are not adequate to predict accurate vibration characteristics of low aspect ratio and higher frequencies. For this reason, the two-dimensional model which is the rectangular plate can be used. In spite of the need, a few research papers on the plate formulation can be found as much detail as the beam formulation. Plate model has been used by a number of researchers. Yoo and Kim [4], using the stretch deformation terms instead of the conventional axial deformation terms, derived linear equations of motion that govern the flapwise bending motion of a rotating plate. The effects of the dimensionless parameters on the modal characteristics of rotating plates investigated. Yoo and Pierre [5] set up a linear discrete dynamic model of a rotating rectangular cantilever plate and calculated the modes of the plate. They studied the dynamic stiffening of the plate by a numerical method. Free vibration of the blade are developed by Sinha and Turner [6]. The governing partial differential equation of motion for the transverse deflection is derived. The achieved solution includes the effect of warping of the cross-section of the blade. The other related articles on rotating plates have been reported in the literature by various researchers such as Farhadi [7], Sinha [8] and Sun [9].

The majority of the above investigations and same articles is mainly restricted to the lateral vibration of plate. In comparison with the out-of-plane vibration analysis, the amount of research devoted to in-plane free vibration behavior of rectangular isotropic and orthotropic plates is extremely small in the literature. However, in addition to lateral vibration, a plate can also undergo in-plane vibration. The lack of published articles regarding in-plane

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vibration problems is probably due to the fact that the natural frequencies of plates in lateral vibration are generally much lower than the fundamental in-plane natural frequency. However, in-plane dynamic response may play prominent role from a design point of view, such as they can be of very practically important in aerospace structures, which operate at very high speeds.

In-plane vibrations of plate usually involve two independent displacement variables. The two displacement components are eventually coupled together through the boundary conditions at an edge. In this regard, the in-plane vibrations may be considered more complicated than their flexural counterpart, as indicated by the mode shapes even for some of the simplest boundary conditions [10]. Bardell et al. [11] studied the in-plane vibration of isotropic rectangular plates having either free or fully constrained boundaries. The results were presented for the lower six modes at two particular aspect ratios. Dozio [12] presented the accurate upper-bound solutions for free in-plane vibrations of single-layer and symmetrically laminated rectangular composite plates with an arbitrary combination of clamped and free boundary conditions. In-plane natural frequencies and modes shapes are calculated and influence of fiber orientation, stacking sequence, aspect ratio and boundary conditions upon the in-plane vibration behavior are also discussed. Free in-plane vibration analysis of orthotropic plates with elastically restrained boundaries was provided by Zhang et al. [13]. Mohazzab and Dozio carried out free in-plane vibration analysis of isotropic plates with skew geometry [14]. The result with various skew angles, aspect ratios, and boundary conditions was presented by them. The free in-plane vibration analysis of orthotropic rectangular plates with non-uniform boundary conditions and internal line supports is performed by Shi et al. [15]. They focused on the effects of boundary restraints on eigenvalues and eigenvectors. The above-stated studies have been carried out on in-plane vibration of stationary rectangular plate. In comparison to non-rotating plate, it appears that the amount of information available for the in-plane vibration characteristics of rotating rectangular plate is extremely limited. However, various practical problems involve in-plane vibration of rotating plates and the dependence of the natural frequencies on rotation speed is typical not only for bending vibrations but also for in-plane vibrations. To the best of author's knowledge, the topic is discussed by Hashemi et al. [16], where a finite element formulation is adopted to study the rotating thick isotropic plates and in-plane vibration and buckling behavior is presented.

It is found that due to the nature of the in-plane problem and its elevated natural frequencies, the analysis of such problems by the finite element method becomes difficult [17]. Furthermore, numerical methods needs much effort in modeling the geometry, which sometimes are not needed especially in the early stage of the design process. In these cases, an approximate, quick, and easy to apply solution is beneficial. Exact solutions of free in-plane vibration of rectangular plates were presented mainly for a few edge boundary conditions, such as plates with at least two opposite edges simply supported [18,19]. For other types of boundary conditions, the efforts were devoted to develop approximate analytical approaches. In the non-rotating problems, analytical type solutions that have been employed by researchers are Ritz, superposition, Fourier series and the Kantorovich method. Singh and Muhammad [20] and Roland et al. [21] analyzed the in-plane motion of plates according to the Ritz energy method. Gorman [17,22] introduced the superposition method to predict in-plane natural frequencies and mode shapes of isotropic and orthotropic plates. Accurate results were presented for rectangular plates with completely free edges, fully clamped edges. Du et al. [10,23] developed Fourier series method for analysis of plates with uniform elastically restrained and point supported edges. Convergence and accuracy of the approaches have been demonstrated through examples. The Kantorovich method was employed by Wang et al. [24] to study isotropic plates with three edges clamped and one edge free, two parallel edges clamped and the opposite edges free and all edges clamped. Applications of this method are addressed in many publications such as bending, buckling and vibration analysis [25–28].

As the review demonstrates, no work and analytical-type solutions are available to study the free in-plane vibrations of rotating rectangular orthotropic plates with cantilever boundary conditions. The mentioned reason motivated us to investigate in this subject that is the main contribution of the current paper. The Hamilton's principle is applied to derive the governing equations of motion and Coriolis effects due to rotation are considered in formulation. We reveal some notable features in isotropic and orthotropic rotating plates and discuss on their eigenfrequency. The influence of different parameters including aspect ratio, thickness ratio, degree of orthotropy and rotation speed upon dimensionless natural frequencies are examined. In order to achieve the above goals, EKM has been adopted due to its conceptual simplicity, less computational effort, wide flexibility, rapid convergency and high accuracy. This method reduces partial differential equation (PDE) to ordinary differential equations (ODE). For the worked out examples, this approximate solution shows good convergence. The formulation is tested on several benchmark problems and reliability of the method is assessed by comparison with available solutions especially non-rotating cases. For rotating cases, the Extended Galerkin Method (EGM) is also applied and the obtained results of two methods are compared with each other. The Campbell diagram of system will be plotted and the resonant frequencies and tune speeds will be obtained and discussed. The mode shape information is important and can be used to improve the design of structures. The natures of in-plane mode shapes may have favorable consequences to monitoring of blade conditions for damage and determining at what location the blade starts to fail. Taking into account these features, the first thorough study, which were not previously reported in this class of problems, presenting the in plane mode shape information for rotating rectangular isotropic and orthotropic plates is done as the dimensionless parameters are varied. Furthermore, the possibility of observing the crossing/veering phenomenon is discussed. Also, the static displacements in the steady state condition are studied and discussed on the validity region of the linear and nonlinear assumptions. The rest of this paper has been organized as follows. The governing equations for in-plane vibration of rotating orthotropic plate as well as the boundary conditions are presented in Section 2, in which non-dimensionalized equations of motion are also attained. Section 3 provides the governing equations by proposed method including the boundary conditions and the details of solution are described. The application of the method to different cases are discussed and analyzed in Section 4. In Section 5, the static displacements due to the centrifugal force are discussed. Finally, the conclusions are drawn in Section 6.

## 2. Mathematical formulation

A schematic diagram of a rotating rectangular plate of orthotropic material is shown in Fig. 1. The one end of this plate is clamped to a rigid hub of radius $R$ and the other edges are free, where $l$, $b$ and $h$ are length, width and thickness respectively; $\Omega$ is the rotation angular velocity. The $xyz$-coordinate system is used in current paper where $x$, $y$ and $z$-axis are respectively along the spanwise, chordwise and thickness direction of plate and the origin of coordinate system is located at the center of clamped edge.