International Journal of Mechanical Sciences 115-116 (2016) 501–515

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

On the free vibrations of size-dependent closed micro/nano-spherical shell based on the modified couple stress theory

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ARTICLE INFO

Article history:
Received 18 February 2016
Received in revised form
17 June 2016
Accepted 6 July 2016
Available online 12 July 2016

Keywords:
Modified couple stress theory
Size-dependency
Fullerene
FSDT
Spherical shell

ABSTRACT

Since classical theory is incapable to justify size-dependency of small scale systems such as micro-electro-mechanics (MEMs) or nano-electro-mechanics (NEMs) systems, modified couple stress theory (MCST) has been developed in order to capture the size effect in the small size investigation. A novel study on the free vibrations of the micro/nano-scale spherical shell based on First-order Shear Deformation Theory (FSDT) and MCST is done. Fullerene (C_{60}) is an appropriate example of spherical micro-scale structures. The governing equations of the modified couple stress spherical shell are derived by using Hamilton's principle. Obtained equations are solved using Generalized Differential Quadrature (GDQ) method. The influences of changing geometrical parameter and scale parameter on the natural frequency are investigated. It is shown that the scale parameter is extremely effective on the natural frequency of the micro/nano-sphere. This issue is bolder in the thick spherical shell. Finally, proper scale parameters are proposed for different nano-scale spheres by comparing numerical results with experimental results.

1. Introduction

Contemporary technological applications need materials that can employ in the micro or nanostructures. In this field, graphene base materials due to their unique mechanical, electrical, and thermal properties attract researcher's attention [1–3]. By introducing Fullerene in 1985, as a carbon base nano-sphere, different kinds of application have been growing, widely. It has a wide range of application in biomedical including intelligent drug delivery, curing cancer and HIV [4], high performance MRI contrast agent and Photodynamic therapy as well as other application in nano machines [5] and electronics [6,7]. Fullerene would be a strong representative of a spherical material in nano/micro-scale. In all of these researches, analyzing dynamic behavior of structures are presented as one of the main steps of designed structures.

There are different methods for dynamic analysis of such structure such as experiment, molecular dynamics simulations, and continuum base methods. Molecular dynamic simulations (MD) consider interatomic and inter-molecular interactions in each step of their analysis which it creates high computational cost, especially when number of atoms increasing. Among modeling methods, continuum based ones are more faster respect to the others and they provide good predictions of general behavior for micro/nano-structures, too. But, experiments show that classical continuum theories are not able to predict material behavior properly. So, size dependent theories have been presented which all of them are based on a real: “deformations of a point of structure are dependent to the deformations of all other points of structures and length scale parameters play major role in behavior of micro/nano-materials”. Therefore, many types of non-classical theories have been developed to overcome this problem. Among these higher order theories, couple stress theory [8–11], strain gradient theory [12], non-local elasticity [13–15] and surface elasticity [16] are more popular among researchers. It should be noted that couple stress theory is a special case of strain gradient theory. Recently, the couple stress theory has been improved by Yang et al. [17] by adding the equilibrium relation of moment of couples to the conventional force equilibrium and moment equilibrium. Consequently, linear isotropic materials only need one new additional parameter-material length scale parameter- beside the classical parameters to consider the size-dependency in micro/nano-scale investigations.

In recent years, both modified strain gradient theories [18] and modified couple stress theory have been extremely utilized in micro/nano-systems to study vibration, bending and buckling of structures such as beams, plates and shells. Park and Gao [19] scrutinized the modified couple stress theory in Bernoulli-Euler

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http://dx.doi.org/10.1016/j.ijmecsci.2016.07.007
0020-7403/

beam in order to analyze static deformation micro cantilever subjected to the point load. In similar way, Kong et al. [20] studied free vibration of a Bernoulli-Euler beam. Ma et al. [21] utilized named theory to investigate static bending and free vibration a Timoshenko beam. They found out that the deflection (similarly rotation) and natural frequency are predicted smaller and higher than classical model results, respectively. Xia et al. [22] established a non-classical Bernoulli-Euler beam based on modified couple stress to investigate free vibration, static deformation and buckling of micro-beams. Also, Free vibration of an embedded micro beam that endures the moving load has been investigated by Simsek [23] as a Bernoulli-Euler beam. In this study, results has been compared with the classical beam theory and conform to the previous consequences. Fu and Zhang [24] considered size-dependency for micro tubules with the Timoshenko beam model. Asghari et al. [25] used the modified couple stress theory for static and dynamic analyze in functionally graded micro-beams. Kahrobayian et al. [26] investigated the behavior of cantilever atomic force microscopes (AFMs) based on the modified couple stress theory. Akgoz and Civalek [27] studied the static deformation and free vibration of higher-order beam based on the strain gradient theory. All the results are compared with other beam theories. Results indicate in smaller slenderness ratio beam, the shear deformation effect is undeniable. Also, it is shown that non-classical theory recognize beam stiffer than corresponding classical beam. Also, researchers such as Reddy [28], Reddy and Arbind [29], Ke et al. [30], Kahrobayian et al. [31], Simsek and Reddy [32], Arbind et al. [33] applied modified couple stress and modified strain gradient theories on functionally graded micro-beams.

Extensive interest of using non-classical elasticity theory has not been only restricted on the beam theories. There are many attempts which done for on the static and dynamic behavior of the low scale plates. Firstly, Tsiatas [34] applied the modified couple stress theory on the Kirchhoff plate model. The proposed model is applicable for micro-plates with complex geometry and different boundary conditions. It is shown that nonlinearity decrease with the increase of material length parameter. Also, decreasing of deflection is concluded with the increasing Poisson's ratio. Yin et al. [35] investigated vibration of micro-plate with non-classical Kirchhoff theory based on the modified couple stress theory. Ma et al. [36] utilized Mindlin plate theory as a special case of first-order shear deformation theory. Both static deformation and natural frequency of the plate have been investigated in this article. Akgoz and Civalek [37] analyzed single layered Graphene sheet by Kirchhoff thin plate model resting on elastic foundation with the same approach. Afterwards, Ke et al. [38] explored on free vibration of Mindlin plate with two different boundary conditions. Shaat et al. [39] modeled a nano-plate, including surface effects based on the modified couple stress theory. Chen and Li [40] continued previous research with Kirchhoff plate for anisotropic elasticity. Also, Salehipour et al. [41] found closed-form answer for free vibration of functionally graded micro/nano-plate. Using a new approach, Wang et al. [42] investigated static deformation of the circular Kirchhoff plate by the MCST.

It is noticeable that all the noted researches have been investigated on Cartesian and rectangular coordinate. Hence, there is no need for changing the formulas of the modified couple stress theory (strain gradient theory). However, changing the coordinate for using MCST in shell investigation is inevitable. Therefore, some probes have been done on curvilinear coordinate. Zhao and Pedroso [43], Guzev and Qi [44], Ashoori and Mahmoodi [45] found modified strain gradient theory (MCST) formulation in general curvilinear coordinates.

Due to complexity of shell theory few researches have been done via MCST. Recently, Zhou and Wang [46] surveyed on the vibration of micro-scale cylindrical shell that conveys fluid based on MCST. One of the most interesting results of this research is shifting the natural frequency by changing the fluid velocity. Zeighampour and Beni [47–49] applied MSCT on cylindrical shell to analyze vibrations and investigating the effect of a length parameter. FSDT has been used to introduce displacement field. Additionally, Zeighampour et al. [50] studied conical shell using FSDT and MCST. Diversions of results by changing the length parameter in the different apex angle have been investigated in this article. Lately, Beni et al. [51] utilized MCST in a cylindrical FGM shell that used FSDT for defining the displacement field. The obtained results show that natural frequencies in MCST are higher than classical theory results.

By comparing literature, it can be easily seen that although MCST and MSGT are trusted methods in micro-scale structure, but lack of investigation of the spherical shell is clearly evident. In this research, free vibrations of spherical shell by using non-classical the modified couple stress theory are studied. First-order shear deformation theory – Sanders type is used to define displacement relations. The governing equations of the modified couple stress spherical shell are derived by using Hamilton's principle. Equations are solved analytically and characteristics, Young's module and Poisson's ratio, of different nano-spheres are used in this path. The shape functions and natural frequencies are exactly assessed. Results are compared with classical theory results, finite element results and experimental results to validate the present method. The size-dependency of shell and change of scale parameter on the natural frequency are investigated. The necessity of using higher-order non-classical theory in small and thick spherical shell has been shown. Behavior of nano-sphere by changing geometrical properties beside the scale parameter is studied and finally, proper values of scale parameter for different nano-spheres are proposed.

2. Mathematical formulations

2.1. Modified couple stress theory

The modified couple stress introduced by Yang et al. [17], expressed strain energy as a function of strain tensor and the gradient of rotation tensor by adding the length parameter to the two classical parameters – Young's module and Poisson's ratio-. It should be noted that length scale parameter shall be obtained from experimental results. Therefore, strain energy of a three-dimensional body that occupies $v$volume in rectangular coordinate is given

$$U = \int_V u dv = \int_V \left(\sigma_{ij}\varepsilon_{ij} + m_{ij}\chi_{ij} \right) dv, \quad i, j = 1, 2, 3 \tag{1}$$

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{2}$$

$$\chi_{ij} = \frac{1}{2}\left(\theta_{i,j} + \theta_{j,i}\right) \tag{3}$$

$$\sigma_{ij} = \lambda\delta_{ij}\varepsilon_{kk} + 2\mu\varepsilon_{ij} \tag{4}$$

$$m_{ij} = 2\mu l^2\chi_{ij} \tag{5}$$

where ε_{ij}, χ_{ij}, σ_{ij} and m_{ij} are strain tensor, symmetric part of rotation gradient tensor, classical stress (Cauchy stress) tensor and higher-order stress tensor. Also, λ and $\mu$are lame' constants and δ_{ij} and l are Kronecker delta and the material length scale parameter, respectively. Moreover, u_i and θ_i stand for displacement vector