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# High order solutions for the magneto-electro-elastic plate with non-uniform materials

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ABSTRACT

In the frame of the scaled boundary finite element method (SBFEM), deformations of the magneto-electro-elastic plate with non-uniform materials are solved in this paper. There are no limitations of shapes and kinds of materials of the plate. At the same time, three different types of external loadings are prescribed at the top plane. Starting with the governing equations of equilibrium and the constitutive equations of magneto-electro-elastic materials, and based on the scaled boundary coordinates, the 3D key partial differential equations are converted into the ordinary differential equations without enforcing the kinematics of plate theory. Only the in-plane dimensions of the plate are divided into 2D finite element, which contributes to reducing the space dimension by one. Moreover, making use of the high order spectral element can produce lumped coefficient matrices, which is helpful to acquire high accuracy and efficiency. The elastic displacements, electric potential and magnetic potential in the vertical direction are solved analytically. The analytical solutions are expressed as a matrix exponent in terms of the thickness which is calculated by the Padé expansion. Finally, several numerical examples are provided to demonstrate the accuracy of the proposed solutions and to display the influence of material partition and loading types on the response of the magneto-electro-elastic plate.



## 1. Introduction

Magneto-electro-elastic (MEE) is a new class of smart composites made of combined piezoelectric and magnetostrictive materials (effects of MEE couplings have been observed in both single-phase materials as well as composites made of piezoelectric, and magnetostrictive phases), which has the ability to covert energy from one form into the other (among magnetic, electric and mechanical energies) and has attracted significant attention of researchers over the past several years on account of encouraging properties for the applications of sensors, actuators, transducers, space structures, sonar applications, ultrasonic imaging devices etc., since the first report on a MEE composite includes piezoelectric phase and piezomagnetic phases by Van den Boomgaard et al. [1]. Because of their inherent multidisciplinary nature the design of devices including smart magneto-electro-elastic composites needs, as a preliminary step, an accurate and effective method to model the electric, magnetic and elastic properties as well as their interactions and couplings. This has motivated a great number of research papers including numerical methods, analytical methods and experiments dealing with the static and dynamic structural-like modelling of magneto-electro-elastic composites.

As far as the structure analysis of magneto-electro-elastic plates is regarded, Pan [2] provided a general method of solving the three-dimensional, anisotropic magneto-electro-elastic, simply supported, and multilayered rectangular plates under both surface and internal static loads. Pan and Heyliger [3] derived an analytical solution for the vibration of three-dimensional, anisotropic, magneto-electro-elastic, and multilayered rectangular plates with simply supported edges. Wang et al. [4] conducted an original exact three-dimensional mixed state space formulation named state vector approach to study the free vibration of piezoelectric, piezomagnetic and elastic hybrid smart laminated plates, which has permitted the reduction of the solution of the assembled system to that of only a third-order one. Similar with the analysis for the corresponding 3D plate problem, which were expressed in terms of the simple quasi-Stroh formalism and the solution in the multilayered plate in terms of the propagator matrix method, Pan and Heyliger [5] have derived an analytical solution for the static bending of an anisotropic, magneto-electro-elastic, and multilayered plate with simply

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supported edges. By using the boundary element method, Ding and Jiang [6] obtained the fundamental solutions for computation of an annular magneto-electro-elastic plate simply-supported on outer and inner surfaces under axial loads. The general solutions in the case of distinct eigenvalues were derived and expressed as five harmonic functions from the governing equations and the strict differential operator theorem. Lage et al. [7] developed and implemented a new layerwise partial mixed finite element model for the analysis of electro-magneto adaptive structures. Chen et al. [8] considered the free vibration problem of simply supported rectangular general transversely isotropic magneto-electro-elastic plates by introducing proper stress and displacement functions for the two independent state equations. Pan and Han [9] presented an exact solution for the multilayered rectangular plate made of functionally graded, anisotropic, and linear magneto-electro-elastic materials with the edges under simply supported conditions and general mechanical, electric and magnetic boundary conditions applied on both the top and bottom surfaces of the plate. Bhangale and Ganesan [10] carried out studies on the free vibration of functionally graded, anisotropic and linear magneto-electro-elastic plates by introducing a semi-analytical finite element method. The state-space method was employed to evaluate the modal parameters of functionally graded, magneto-electro-elastic, and multilayered plates by Chen et al. [11]. Bhangale and Ganesan [12] performed a semi-analytical finite element method to the static analysis of functionally graded, anisotropic and linear magneto-electro-elastic plates. Natural frequencies of orthotropic magneto-electro-elastic graded composite plates were determined using a discrete layer model with two different approaches by Ramirez et al. [13]. Ramirez et al. [14] adopted an approximate solution based on using a discrete layer model for the free vibration problem of two-dimensional magneto-electro-elastic laminates to determine their fundamental behaviour. Assuming that the shape functions were used in the plane of plate and one dimensional finite element were used across the thickness of the plate, Annigeri et al. [15] studied the free vibration of simply supported multiphase magneto-electro-elastic plates by a semi-analytical finite element method. Chen et al. [16] presented an analytical treatment using the state-vector approach for the propagation of harmonic waves in magneto-electro-elastic multilayered plates, where the general anisotropic and three-phase coupled constitutive equations were used. Feng and Su [17] investigated the dynamic anti-plane problem for a functionally graded magneto-electro-elastic plate containing an internal or an edge crack parallel to the graded direction. Chen et al. [18] proposed the state-vector approach to analyze the free vibration of magneto-electro-elastic laminate plates. Feng and Pan [19] analyzed the anti-plane problem for an interfacial crack between two dissimilar magneto-electro-elastic plates subjected to anti-plane mechanical and in-plane magneto-electrical impact loadings. Li et al. [20] considered the problem of a functionally graded, transversely isotropic, magneto-electro-elastic circular plate acted on by a uniform load. Wu et al. [21] presented an analytical treatment for the propagation of harmonic waves in magneto-electro-elastic multilayered plates, where the general anisotropic and three-phase coupled constitutive equations were used. Liu et al. [22] obtained an exact solution for thickness-extension vibrations of a magnetoelectricity plate with thick electrodes of unequal thickness. Simões et al. [23] developed a finite element model based on a higher-order plate theory for the static and free vibration analysis of magneto-electro-elastic plates. This model allowed obtaining in static analysis the through-thickness distributions of primary variables mechanical displacements, electric potentials and magnetic potentials as well as for electric displacements and magnetic inductions. Phoenix et al. [24] addressed an extension of the RMVT for the static and dynamic analysis of the magneto-electro-elastic plate problem. Wang et al. [25] established the state variable formulation for free vibration of magneto-electro-elastic hybrid laminated plates. Liu and Chang [26] derived a closed form expression for the transverse vibration of a magnetoelectroelastic (MEE) thin plate. Liu [27] obtained an exact deformation solution for the magneto-electro-elastic fibre-reinforced thin plate. Based on linear three-dimensional theory of elasticity coupled with magnetic and electric fields, Wang et al. [28] analyzed axisymmetric bending of functionally graded circular magneto-electro-elastic plates of transversely isotropic materials. Biju et al. [29] studied the dynamic response of moderately thick magneto-electro-elastic plate using magnetic vector potential in finite element formulation. Yu et al. [30] investigated the dispersion behaviour of waves in a layered magneto-electro-elastic plate. Based on an equivalent single-layer model, Alaimo et al. [31] presented an isoparametric four-node finite element for multilayered magneto-electro-elastic plates. Zhong et al. [32] developed a magneto-electro-elastic Reissner–Mindlin model for the heterogeneous multi-layered laminates made of functionally graded magneto-electro-elastic material using the variational asymptotical method. Chang [33] carried out the investigation of the vibration characteristics of transversely isotropic magneto-electro-elastic (MEE) rectangular plates in contact with fluid, and he [34] further investigated the natural frequencies of a transversely isotropic magneto-electro-elastic (MEE) rectangular thin plate which is in contact with fluid. Chang [35] also conducted the study of the free vibration, deterministic vibration and random vibration characteristics of transversely isotropic Magneto-Electro-Elastic (MEE) rectangular plates in contact with fluid. Kattimani and Ray [36] proposed an analysis of the active constrained layer damping (ACLD) of large amplitude vibrations of smart magneto-electro-elastic (MEE) plates. Milazzo [37] provided layer-wise and equivalent single layer plate models for magneto-electro-elastic multiphysics laminates in a unified framework. Milazzo [38] derived a model for the large deflection analysis of magneto-electro-elastic laminated plates. Chen et al. [39] developed a semi-analytical discrete-layer model of the governing differential equations which was applied to typical layered MEE media under combined clamped and free lateral boundary conditions. Xie and Chi [40] studied the dynamic response sensitivity of a simply supported functionally graded magneto-electro-elastic plates by combining the analytical method with finite element method. Li et al. [41] presented thermo-magneto-electro-elastic field in a heterogeneous circular plate subjected to thermal loadings uniformly distributed on the main boundaries. Alaimo et al. [42] achieved an original finite element formulation for the analysis of large deflections in magneto-electro-elastic multi-layered plates. Razavi and Shooshtari [43] studied the nonlinear free vibration of symmetric magneto-electro-elastic laminated rectangular plates with simply supported boundary condition. Based on the thin plate theory along with the von Karman's nonlinear strains, nonlinear free and forced vibration of a transversely isotropic rectangular magneto-electro-elastic thin plate with simply supported boundary conditions is analyzed by Shooshtari and Razavi [44]. Based on the three-dimensional elasticity theory, Xin and Hu [45] derived semi-analytical solutions for free vibration of simply supported and multilayered magneto-electro-elastic plates applying a newly developed hybrid analysis, which combines the state space approach (SSA) and the discrete singular convolution (DSC) algorithm. Milazzo [46] used a variable kinematics approach for moderately large deflection analysis of smart magneto-electro-elastic multilayered plates. Kattimani and Ray [47] dealt with the analysis of active damping of geometrically nonlinear vibrations of functionally graded magneto-electro-elastic (FGMEE) plates integrated with the patches of the active constrained layer damping (ACLD) treatment. Shooshtari and Razavi [48] studied analytically the nonlinear free vibration of symmetrically laminated magneto-electro-elastic rectangular plate resting on an elastic foundation. Farajpour et al. [49] developed a non-local continuum model for the