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Arc-shaped permeable interface crack in an electrostrictive fibrous composite under uniform remote electric loadings

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ABSTRACT

We present a rigorous solution of the problem of an arc-shaped interface crack embedded between a circular electrostrictive fiber and a foreign matrix subjected to uniform remote electric loadings. The crack faces are assumed to be permeable to an electric field. Mode-I and Mode-II stress intensity factors for the oscillatory singular stress field at the crack tips are obtained using complex variable methods. We find that the stress intensity factors typically increase as the fiber becomes harder. For certain combinations of the arc length of the crack and material constants of the fiber-matrix system, we show that remote electric loadings may actually prevent mode-I interfacial fracture. In particular, we show that when the fiber-matrix system degenerates to a homogeneous electrostrictive material and the medium inside the crack is identical to that surrounding the material remotely, the application of remote electric loadings continue to have a significant influence on both mode-I and mode-II fracture.

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1. Introduction

Electrostriction is a property of all dielectric materials that causes them to deform under the application of an electric field [1–3]. In contrast to piezoelectricity, electrostriction is almost exclusively a nonlinear effect (see, for example, [4–7]), especially in most electroactive polymers although electrostriction in most dielectrics is much weaker than the piezoelectric effect in general piezoelectric materials. In recent years, however, new electrostrictive materials (for example, PMN-matrix ceramics) have been identified for which electrostriction is comparable with general piezoelectricity. As is the case with the majority of piezoelectric materials, however, most efficient electrostrictive materials are brittle and susceptible to cracking as a result of damage accumulation through sustained fatigue. In an effort to predict the reliability of electrostrictive devices, fracture problems involving electrostrictive materials weakened by cracks have attracted a great deal of attention in the literature. For example, McMeeking [8] discussed the electrically-induced stress concentration around a crack-like flaw embedded in an electrostrictive material for various aspect ratios of the flaw. Yang and Suo [9], Hao et al. [10] and Gong and Suo [11] studied the electrostrictive cracking around electrode tips embedded in a single layer or multilayer ceramic actuator. In addition to [9–11], Ru et al. [12] analyzed the interfacial cracking between electrodes and a surrounding ceramic matrix and subsequently verified that interfacial cracking as opposed to the general matrix cracking studied in [9–11] is indeed the dominant failure mechanism in electrostrictive multilayer actuators. Beom et al. [13] performed asymptotic analyses of a crack within an electrostrictive material subjected to electric loadings and found that the size of the stress intensity factor-dominant region is much smaller than the electric saturation zone. This further suggests that the fracture criterion for general elastic materials under mechanical loadings may still be valid for electrostrictive materials under electric loadings.

In [5], McMeeking and Landis pointed out that the space surrounding the electrostrictive material should form part of any analysis since the electric field exists also in that space and the corresponding Maxwell stresses may indeed induce tractions on the edge of the material. This important piece of information was not considered in any of the aforementioned studies [8–13]. However, in [14], Gao et al. did address this deficiency in the case of a single rectilinear crack in an electrostrictive material surrounded by a certain medium. Subsequently, Zheng and Gao [15] extended the investigation to incorporate the contribution of the surrounding space in the case of an arc-shaped crack inside a homogeneous electrostrictive material. Unfortunately, in [15], the authors assumed the electric field inside the crack to be uniform which is incompatible with the exact electric conditions on the crack faces and may well lead to significant errors in the determination of the stress intensity factors at the crack tips.

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In response to the deficiencies in [15], we consider here the more general and indeed practical case of an arc-shaped interface crack in an electrostrictive fibrous composite subjected to uniform remote electric loading in which the exact (non-uniform) electric field inside the interface crack is taken into account. We assume that the crack is permeable to an electric field, a reasonable assumption based on the results established by McMeeking in [8]. Indeed, McMeeking pointed out that the assumption of impermeability of a slender hole is physically unacceptable unless the aspect ratio of the hole (thickness to length) is significantly larger than the ratio of the dielectric constant of the interior of the hole to that of the surrounding material, suggesting therefore that the hole should be taken as permeable in the limit as the hole approaches the configuration of a crack.

The paper is organized as follows. In Section 2, the problem under consideration is formulated based on a simplified linear dielectric response. In Sections 3 and 4, we use complex variable methods to establish closed-form solutions for the electric and stress fields in the case of an arc-shaped interface permeable crack. In Sections 5 and 6, analytical and numerical results for the stress intensity factors at the crack tips are presented. Finally, our main findings are summarized in Section 7.

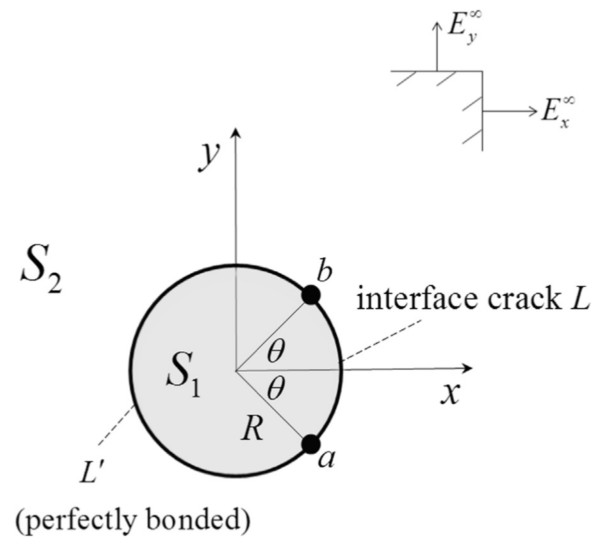

Fig. 1. Arc-shaped interface crack in a fibrous electrostrictive composite.

2. Basic equations and problem description

2.1. Basic equations for isotropic electrostrictive materials

We consider a body composed of an isotropic electrostrictive material in the absence of any piezoelectricity and free charge density inside the body. In a Cartesian coordinate system (x_1, x_2, x_3), assuming the usual convention of summation over repeated indices (in this Section *only*) and denoting the components of mechanical stress, strain, displacement, electric displacement and electric field by σ_{kl}, e_{kl}, u_l, D_l and E_l (k, l=1, 2, 3), respectively, we can describe the equilibrium equations for the material as

$$\partial D_l/\partial x_l = 0, \tag{1}$$

$$\partial\tilde{\sigma}_{kl}/\partial x_l = 0, \quad \tilde{\sigma}_{kl} = \sigma_{kl} + \sigma_{Mkl}, \tag{2}$$

where $\tilde{\sigma}_{kl}$ and σ_{Mkl} denote the Cartesian components of the total stress and Maxwell stress, respectively. In particular, the Maxwell stress can be expressed in terms of the electric field components as

$$\sigma_{Mkl} = \varepsilon\left(2E_kE_l - E_jE_j\delta_{kl}\right)/2, \tag{3}$$

where ε is the dielectric constant, j=1, 2, 3 and δ_{kl} represents the Kronecker delta. On the other hand, the constitutive equations for the material are given by [16]

$$\sigma_{kl} = \lambda e_{jj}\delta_{kl} + 2\mu e_{kl} - \left(\alpha E_kE_l + \beta E_jE_j\delta_{kl}\right)/2, \tag{4}$$

$$D_k = (\varepsilon\delta_{kl} + \alpha e_{kl} + \beta e_{jj}\delta_{kl})E_l, \tag{5}$$

where α and β are two independent electrostrictive coefficients, while λ and μ are the Lame constants. The mechanical and electric variables are coupled in Eqs. (4) and (5) making it extremely difficult to obtain analytical results in any theory which uses these equations, in particular, in fracture of electrostrictive materials. Consequently, to obtain some approximate results for crack problems of electrostrictive materials, a simplified linear dielectric response has been used extensively in the literature (see, for example, [8–15]) instead of the original Eq. (5). That is

$$D_k = \varepsilon E_k, \tag{6}$$

which essentially uncouples the mechanical and electric variables. In the remainder of the paper, we will make the same assumption and adopt Eq. (6) for our specific problem.

2.2. Problem details

As shown in Fig. 1, we consider a circular electrostrictive fiber embedded in an infinite foreign electrostrictive matrix subjected to plane-strain deformations with uniform electric fields E_x^∞ and E_y^∞ applied at infinity. An interface crack L (emanating from point a and ending at point b: see Fig. 1) whose faces are assumed to be permeable to the electric field originates from the partial debonding at the fiber-matrix interface. In Section 2.1, the basic equations describing an electrostrictive material were introduced using a Cartesian coordinate system denoted in the plane by (x_1, x_2). At this stage it becomes necessary to make a small change in notation to accommodate the fact that the particular plane problem under consideration involves the indices 1 and 2 in the description of the two distinct material regions S_1 and S_2 (see Fig. 1). Consequently, henceforth, we replace the notation (x_1, x_2) with (x, y) to denote the same coordinate system (as shown in Fig. 1). We remind the reader that summation over repeated indices was restricted to Section 2.1 above and so does not apply in what follows and in the remainder of the paper.

Following the simplified linear dielectric response given by Eq. (6), the electric field (E_x, E_y), the electric displacement (D_x, D_y), the Maxwell stress (σ_{Mxx}, σ_{Myy}, σ_{Mxy}), the mechanical stress (σ_{xx}, σ_{yy}, σ_{xy}), the total stress (the sum of Maxwell stress and mechanical stress; $\tilde{\sigma}_{xx}$, $\tilde{\sigma}_{yy}$, $\tilde{\sigma}_{xy}$) and the displacement (u_x, u_y) of the fiber-matrix system can be described in terms of the complex functions $f(z)$, $\varphi(z)$ and $\psi(z)$ by [16]

$$E_x^{(j)} - iE_y^{(j)} = -f_j'(z), \quad (j = 1, 2) \tag{7}$$

$$D_x^{(j)} - iD_y^{(j)} = -\varepsilon_j f_j'(z), \quad (j = 1, 2) \tag{8}$$

$$\left.\begin{aligned} &\sigma_{Myy}^{(j)} - \sigma_{Mxx}^{(j)} + 2i\sigma_{Mxy}^{(j)} = -\varepsilon_j\left[f_j'(z)\right]^2, \\ &\sigma_{Mxx}^{(j)} + \sigma_{Myy}^{(j)} = 0, \quad (j = 1, 2) \end{aligned}\right\} \tag{9}$$

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