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A global formulation for complex rod structures in isogeometric analysis

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Linear elasticity analysis of complex rod structure in the framework of isogeometic analysis is researched. An efficient global formulation for spatial Timoshenko rod structure is proposed, in which the general geometric form of complex rod structure with arbitrary curvatures and torsions described by multi-patches is adopted. The Frenet–Serret formula is used for the derivatives of direction vector with respect to the arc length of the rod, and a concise local strain formula with respect to the global control variables is obtained. The element stiffness matrices are calculated in global coordinate system therefore they can be assembled directly, which makes present formulation suitable for multi-patch connection. The performance with respect to the number of quadrature points are tested, and the results indicate that reduced integration schemes could be adopted to improve the performance. The effectiveness of proposed formulation is verified by numerical analyses of complex structures. Comparison with the local form formulation shows that the proposed formulation has less computational cost for multi-patch simulation, and is easier to be post-processed as well.

1. Introduction

Spatial rod structures play important roles in many applications. In traditional design of steel structures in civil engineering, straight rods are widely used. In present innovation design, more and more components with complex spatial shapes are applied, for example in the design of Beijing National Stadium (Bird's nest). In biomechanics the simulation of DNA structures greatly relies on the approximate model of double helix curved rod structures.

The numerical simulations of rod structures are used widely in the structure design, analysis and optimization. Finite element analysis (FEA) for the simulation of rods has been developed for a long time. As is known to all that in FEA the complex structures are discreted into straight rod elements. However, for spatial rod structures with curvatures and torsions, the approximation in geometry can increase the computational price and has significant influence on the computational precision. To overcome this problem, Arunakirinathar and Reddy introduced a mixed finite element formulation for spatial curved elastic rods in 1993 [1]. Choit and Lim proposed a plane curved Timoshenko rod formulation on the assumed strain fields in the framework of FEA in 1995 [2]. Taktak et al. presented a helical beam element based on a mixed-hybrid formulation in 2005 [3].

Since 2005, isogeometric analysis (IGA) [4] has been proving itself a suitable solution to the simulation of complex structures. In IGA the complex geometry can be exactly described, thus the problem turns as to find a suitable formulation for real-world spatial frameworks. At present, the most popular type of formulation for curved Timoshenko beams or rods in IGA is the one using local strain formula with respect to the local control variables. This type of formulation is described as "local form" hereinafter. A local form Timoshenko curved beam formulation was introduced in [5] and the performance of some unlocking methodologies such as the discrete strain gap method, the B-bar method and the selective and reduced integration method were studied. In [6], an improved numerical integration rule for membrane and shear locking was proposed in the context of the local form formulation. In 2015, Luu et al. had done excellent works on bending, buckling [7] and vibration analysis [8] of laminated of laminated curved beams, and also the vibration analysis of Timoshenko beams [9]. Since 2015, some numerical analysis tools for masonry arches were developed [10,11]. Recently, Hu et al. [12] developed the order reduction method to obtain an excellent locking-free isogeometric formulation. However, these kinds of beam and rod formulations and related studies are mainly focused on single patch geometry, and the formulation of real complex rod structures still needs further study.

In this paper, the linear elastic simulation of complex spatial rod structures by isogeometric approach is performed. The rod structure is described by exact geometry model with multi-patch.

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The full detail spatial form of rods are described by Non-Uniform Rational B-Spline (NURBS) curves. Both in plane curvature and out plane torsion are considered in the strain-displacement formulation. A global rod formulation, which is concise in element stiffness formation and straightforward in simulation of complex spatial rod structure is established.

2. Global beam formulation

2.1. Geometric description

The geometry of the spatial rod as shown in Fig. 1 is described by its mid-line $\mathbf{c}(x, y, z)$, which can be defined precisely by

$$x(\xi) = \sum_{i=1}^{n} R_i(\xi)x_i, \quad y(\xi) = \sum_{i=1}^{n} R_i(\xi)y_i, \quad z(\xi) = \sum_{i=1}^{n} R_i(\xi)z_i \tag{1}$$

where (x_i, y_i, z_i) are coordinates of the i-th control points, and R_i are the NURBS basis functions [13,14]. Compared with general bars, and also elements in commercial FEM softwares, present element has the unique advantage that it is NURBS based, thus it can suit curves with arbitrary curvatures and torsions.

The relation between infinitesimal arch length $\mathrm{d}s$ and the parametric coordinate $\mathrm{d}\xi$ is

$$\mathrm{d}s = J(\xi)\mathrm{d}\xi \tag{2}$$

and

$$J(\xi) = \sqrt{\left(\dot{x}(\xi)\right)^2 + \left(\dot{y}(\xi)\right)^2 + \left(\dot{z}(\xi)\right)^2} \tag{3}$$

where the superscript $\dot{v}(\xi)$ means $\mathrm{d}v/\mathrm{d}\xi$.

The Frenet frame $(\mathbf{t}, \mathbf{n}, \mathbf{b})$ is employed to define the local coordinate system (Fig. 1). The unit tangent vector $\mathbf{t}(t_x, t_y, t_z)$, normal vector $\mathbf{n}(n_x, n_y, n_z)$ and the binormal vector $\mathbf{b}(b_x, b_y, b_z)$ are calculated by

$$\mathbf{t}(\xi) = \frac{\dot{\mathbf{c}}(\xi)}{|\dot{\mathbf{c}}(\xi)|}, \quad \mathbf{n}(\xi) = \frac{\dot{\mathbf{c}}(\xi) \times \ddot{\mathbf{c}}(\xi)}{|\dot{\mathbf{c}}(\xi) \times \ddot{\mathbf{c}}(\xi)|}, \quad \mathbf{b}(\xi) = \mathbf{t}(\xi) \times \mathbf{n}(\xi) \tag{4}$$

respectively in which $\dot{\mathbf{v}}$ means the derivatives of the components of vector $\mathbf{v}$ with respect to ξ, $|\mathbf{v}|$ means the module length of vector $\mathbf{v}$, $(\mathbf{v_1} \times \mathbf{v_2})$ means the vector product of vectors $\mathbf{v_1}$ and $\mathbf{v_2}$.

For a given parameter coordinate ξ, the curvature κ and torsion τ of the rod are

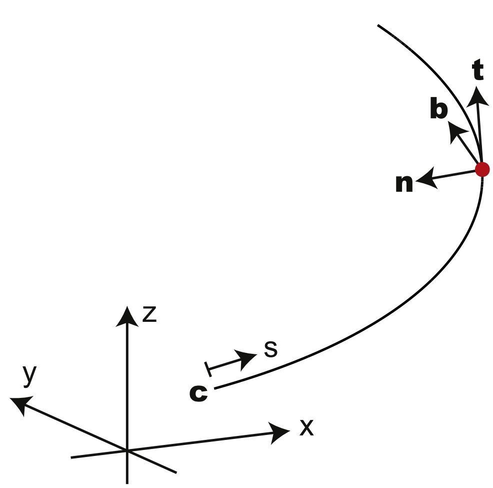

Fig. 1. A spatial rod and its Frenet frame at a given point. Two sets of coordinate systems, meaning the global system and the local system varying along the rod are defined.

$$\kappa(\xi) = \frac{|\dot{\mathbf{c}}(\xi) \times \ddot{\mathbf{c}}(\xi)|}{|\dot{\mathbf{c}}(\xi)|^3}$$
$$\tau(\xi) = \frac{(\dot{\mathbf{c}}, \ddot{\mathbf{c}}, \dddot{\mathbf{c}})}{(\dot{\mathbf{c}} \times \ddot{\mathbf{c}})^2} \tag{5}$$

here $(\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3})$ means the mixed product, and it is noted that $\kappa \geq 0$ stands everywhere because the module is always non-negative.

The Frenet–Serret formula is used in our formulation

$$\begin{bmatrix} \dfrac{\mathrm{d}\mathbf{t}(\xi)}{\mathrm{d}s} \\ \dfrac{\mathrm{d}\mathbf{n}(\xi)}{\mathrm{d}s} \\ \dfrac{\mathrm{d}\mathbf{b}(\xi)}{\mathrm{d}s} \end{bmatrix} = \begin{bmatrix} 0 & \kappa(\xi) & 0 \\ -\kappa(\xi) & 0 & \tau(\xi) \\ 0 & -\tau(\xi) & 0 \end{bmatrix} \begin{bmatrix} \mathbf{t}(\xi) \\ \mathbf{n}(\xi) \\ \mathbf{b}(\xi) \end{bmatrix} \tag{6}$$

For detailed derivation of these formulas refer to books of computational geometry, such as Ref. [15].

2.2. Strain formula

The concept of the rod element is defined according to Fig. 2. There are 6 degrees of freedom (DOFs) for one node (control point), which are set in accordance with the Frenet frame. The local strain formula derived from the local control variables is given by [2]:

$$\begin{bmatrix} \varepsilon_t \\ \gamma_n \\ \gamma_b \\ \eta_t \\ \chi_n \\ \chi_b \end{bmatrix} = \begin{bmatrix} \dfrac{\mathrm{d}u_t}{\mathrm{d}s} - \kappa u_n \\ \dfrac{\mathrm{d}u_n}{\mathrm{d}s} + \kappa u_t - \tau u_b - \theta_b \\ \dfrac{\mathrm{d}u_b}{\mathrm{d}s} + \tau u_n + \theta_n \\ \dfrac{\mathrm{d}\theta_t}{\mathrm{d}s} - \kappa\theta_n \\ \dfrac{\mathrm{d}\theta_n}{\mathrm{d}s} + \kappa\theta_t - \tau\vartheta_b \\ \dfrac{\mathrm{d}\theta_b}{\mathrm{d}s} + \tau\theta_n \end{bmatrix} \tag{7}$$

Here the κ and τ are obtained by Eq. (5). The formulation using Eq. (7) is called the local form formulation, and it is not suitable for multi-patch calculation, which would be discussed in Section 3.

By Eq. (7), the local strain formula with respect to the global control variables is derived. $[\tilde{u}] = \left[u_t \ u_n \ u_b\right]^{\mathrm{T}}$ and $[\tilde{\theta}] = \left[\theta_t \ \theta_n \ \theta_b\right]^{\mathrm{T}}$ are defined as displacement and rotation fields in local coordinates. Looping over all quadrature points in current element, and at arbitrary quadrature point ξ_i, these local fields can be equally described in global coordinate system as $[u] = \left[u_x \ u_y \ u_z\right]^{\mathrm{T}}$ and $[\theta] = \left[\theta_x \ \theta_y \ \theta_z\right]^{\mathrm{T}}$, with transformation through rotation $[u] = [R_i][\tilde{u}]$, $[\theta] = [R_i][\tilde{\theta}]$. The rotation matrix $[R_i]$ is indicated as

$$[R_i] = \begin{bmatrix} t_x & n_x & b_x \\ t_y & n_y & b_y \\ t_z & n_z & b_z \end{bmatrix}_{\xi_i} \tag{8}$$

Substituting Eq. (8) into Eq. (7), and the derivatives of components of vectors $\mathbf{t}$, $\mathbf{n}$, $\mathbf{b}$ with respect to the arc length of the rod s are calculated by the Frenet–Serret formula in Eq. (6), the local strain formula with respect to the global variables is obtained as follows: