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Free and forced vibrations of shear deformable functionally graded porous beams

Da Chen [a], Jie Yang [b,*], Sritawat Kitipornchai [a]

[a] School of Civil Engineering, the University of Queensland, St Lucia, Brisbane 4072, Australia
[b] School of Engineering, RMIT University, PO Box 71, Bundoora, VIC 3083, Australia

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ABSTRACT

This paper investigates the free and forced vibration characteristics of functionally graded (FG) porous beams with non-uniform porosity distribution whose elastic moduli and mass density are nonlinearly graded along the thickness direction. Both symmetric and asymmetric porosity distributions are considered. The relationship between coefficients of porosity and mass density is determined based on the typical mechanical property of an open-cell metal foam. Within the framework of Timoshenko beam theory to include the effect of transverse shear strain and by employing Lagrange equation method together with Ritz trial functions, the equation of motion is derived then solved by using Newmark-β method in the time domain. Natural frequencies and transient dynamic deflections are obtained for porous beams under different loading conditions, including a harmonic point load, an impulsive point load and a moving load with constant velocity. A detailed numerical study is conducted to examine the effects of varying porosity distribution, porosity coefficient, slenderness ratio and boundary condition, shedding important insights into the design of functionally graded porous beams to achieve improved dynamic behavior.

1. Introduction

As one of the most important developments in new advanced composite materials, functionally graded materials (FGMs) have attracted immense attention from research and engineering communities since they were first proposed by Japanese scientists [1]. FGMs are inhomogeneous composite materials containing two or more constituents whose material composition continuously and smoothly varies along certain direction(s). Compared with conventional homogeneous materials, mechanical properties of FGMs can be appropriately tailored for design purpose [1,2].

Lightweight structures made of porous materials, such as metal foams, provide unique potential for wide applications in aerospace, automotive and civil engineering [3–6]. With the excellent energy-absorbing capability, metal foams are also regarded as one of the most promising candidates for structures under dynamic loading [7–10]. Combining FGMs with porous materials leads to the development of novel FG porous structures characterized by the graded distribution of internal pores in the microstructure, enabling the local density to be a design variable to achieve improved structural performance.

Most of current studies are focused on the stability and bending problems of FG porous beams and plates. Magnucki and Stasiewicz [11] obtained an explicit expression for the critical load of FG beam with graded porosity by using the principle of stationarity of the total potential energy. Magnucka-Blandzi [12] investigated the problem of axi-symmetrical deflection and buckling of circular porous–cellular plate with the geometric model of nonlinear hypothesis. Grygorowicz et al. [13] studied the elastic buckling of three-layered beam consisting of metal foam core with varying mechanical properties, and applied a broken line hypothesis and a nonlinear hypothesis to the displacement field, respectively. Jabbari et al. [14] presented the buckling analysis of a solid circular plate under radial loading made of FG porous materials based on the geometrical nonlinearities in the Love–Kirchhoff hypothesis and Sanders nonlinear strain–displacement relationship. Mojahedin et al. [15] carried out the study of thermal and mechanical stability of solid circular plates made of saturated/unsaturated FG porous materials with piezoelectric actuators by employing the energy method and classical plate theory. Chen et al. [16] gave the elastic buckling and static bending solutions of shear deformable FG porous beams based on Timoshenko beam theory and Ritz method. Their research was focused on the influence of varying porosity distribution on the structural performance. The energy-absorbing capability of FG porous structures has also been discussed, see, for example, those in Refs. [17–21].

* Corresponding author. Tel.: +61 03 99256169; fax: +61 03 99256108.
E-mail address: j.yang@rmit.edu.au (J. Yang).

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Understanding the dynamic behavior of FG beams and plates is crucial for their engineering applications. Ke et al. [22] gave the nonlinear free vibration solutions of FG nanocomposite beams reinforced by single-walled carbon nanotubes within the framework of Timoshenko beam theory and von Kármán type displacement–strain relationship. Yang et al. [23] investigated the free and forced vibrations of slender FGM beams under an axial compressive force and a concentrated transverse moving load, and used the rotational spring model to consider the sectional flexibility caused by open edge cracks. Şimşek and Kocatürk [24] studied the free vibration characteristics and transient deflections of a simply supported FG beam subjected to a concentrated moving harmonic load by employing Lagrange equation method within the framework of Euler–Bernoulli beam theory. Khalili et al. [25] proposed a mixed Ritz–DQ method to analyze the dynamic behavior of FG beams carrying moving loads with Ritz method to discretize the spatial partial derivatives and DQ method to discretize the temporal derivatives. Bodaghi and Shakeri [26] presented an analytical approach for the free vibration and transient response of simply supported FG piezoelectric cylindrical panel impacted by various blast pulses with Hamilton's principle. Zhang et al. [27] analyzed the effect of open edge crack on the nonlinear dynamics and chaotic behavior of FG plates in thermal environment and subjected to parametric and external excitations based on Reddy's third-order shear deformation plate theory and method of multiple scales.

This paper conducts the free and forced vibration analysis of FG porous beams with symmetric and asymmetric porosity distributions. Timoshenko beam theory is employed to account for the effect of transverse shear strain. The governing equation of motion is derived based on Lagrange equation method and Ritz trial functions. Ritz method in conjunction with Newmark-β method is used in this paper due to their excellent computational efficiency compared with finite element method which usually needs a complicated mesh grid and a very large number of elements to obtain results with reasonable accuracy. Natural frequencies and dynamic deflections of FG porous beams under a harmonic point load, an impulsive point load and a moving load with constant velocity are obtained for beams with four different boundary conditions including hinged–hinged (H–H), clamped–clamped (C–C), clamped–hinged (C–H) and clamped–free (C–F). The effects of different system parameters and porosity distributions are discussed in detail to identify the effective way to improve the dynamic performance of FG porous beams.

2. Functionally graded porous beam

An FG porous beam of thickness h, width b, and length L with two different porosity distributions along the thickness direction is shown in Fig. 1, where the x–z coordinate system is also given with the x-axis in the length direction and the z-axis in the thickness direction. Porosity distribution 1 [11,16] and distribution 2 [14,16] are described by Fig. 1(a) and (b), respectively. The nonlinear and continuous variations in Young's modulus, shear modulus and mass density due to the graded non-uniform porosity can be described by Eq. (1) for distribution 1 and Eq. (2) for distribution 2.

$$\begin{cases} E(z) = E_1\left[1 - e_0 \cos\left(\pi \frac{z}{h}\right)\right] \\ G(z) = G_1\left[1 - e_0 \cos\left(\pi \frac{z}{h}\right)\right] \\ \rho(z) = \rho_1\left[1 - e_m \cos\left(\pi \frac{z}{h}\right)\right] \end{cases} \tag{1}$$

$$\begin{cases} E(z) = E_1\left[1 - e_0 \cos\left(\frac{\pi}{2}\frac{z}{h} + \frac{\pi}{4}\right)\right] \\ G(z) = G_1\left[1 - e_0 \cos\left(\frac{\pi}{2}\frac{z}{h} + \frac{\pi}{4}\right)\right] \\ \rho(z) = \rho_1\left[1 - e_m \cos\left(\frac{\pi}{2}\frac{z}{h} + \frac{\pi}{4}\right)\right] \end{cases} \tag{2}$$

where E_1, G_1, ρ_1 are the maximum values of Young's modulus, shear modulus and mass density, E_2, G_2, ρ_2 are the corresponding minimum values, which can be used to obtain the porosity coefficient $e_0 = 1 - E_2/E_1 = 1 - G_2/G_1$ $(0 < e_0 < 1)$ and the porosity coefficient for mass density $e_m = 1 - \rho_2/\rho_1$ $(0 < e_m < 1)$. The relationship between Young's modulus and shear modulus is $G_i = E_i/[2(1+\nu)]$ $(i = 1, 2)$ where ν is the constant Poisson's ratio across the beam thickness [4,28].

It is evident from Fig. 1 and Eqs. (1) and (2) that porosity distribution 1 is symmetric with the maximum values of elastic moduli and mass density on the top and bottom surfaces, and the minimum values on the mid-plane due to the largest size and density of internal pores, while porosity distribution 2 is asymmetric with the maximum values on the top surface and the minimum values on the bottom surface.

The typical mechanical property of an open-cell metal foam [16,29,30] can be expressed as Eq. (3), which is used to determine

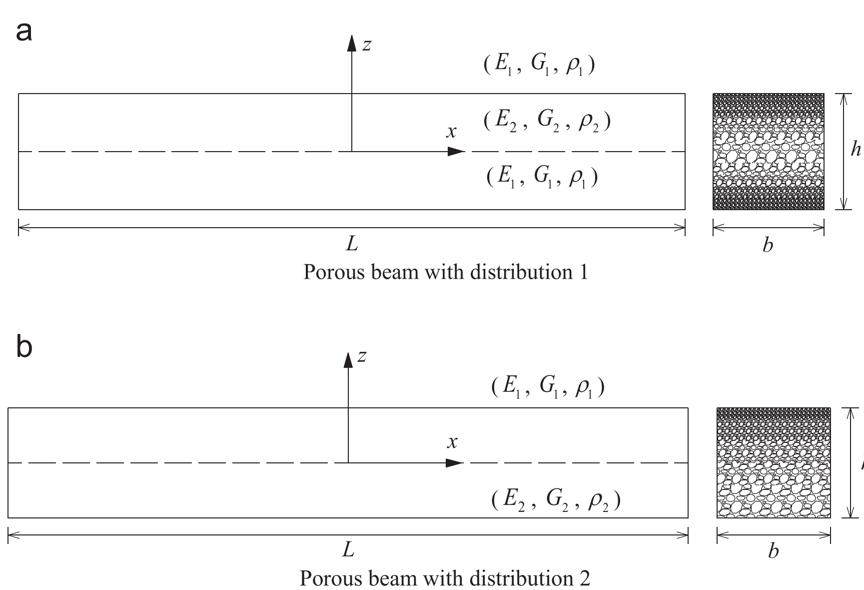

Fig. 1. Functionally graded porous beam with different porosity distributions.