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High frequency vibration of a rectangular micropolar beam: A dynamical analysis

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ABSTRACT

The present paper deals with the dynamical solution of a rectangular micropolar beam in high frequency domain. Analogous to symplectic approach by W. Zhong, a new methodology is adopted to solve the vibrational problem by using Hamiltonian principle with Legendre's transformation. This paper extends the symplectic elasticity approach and dual variables to a different version that simplifies the solution procedure.

1. Introduction

In the analysis of beam theory, semi-inverse method, i.e. finding a trial solution which can satisfy the corresponding boundary conditions, is one of the common methods in most of the engineering practices. Theory of beams and its development has been well discussed by Timoshenko [1]. In this book, a large amount of text are accessible consisting of the solutions of various elasticity problems by using semi-inverse method. In elasticity such an approach is very common because set of governing equations for the elastic system are generally complex in nature.

Semi-inverse method does not lead to complete set of solution for a problem. Trial solutions that are obtained are only applicable to a very specific set of boundary conditions, for example, plates with two opposite sides simply supported. Furthermore, conventional method often generates higher order partial differential equation with a single variable by eliminating other unknown functions. As a result there is no scope to utilize the effective methods in mathematical physics such as separation of variables and expansion of eigen functions.

In 1990, for solving the basic problems in solid mechanics, Zhong (see Ref. [2]) have applied symplectic approach which created a new avenue to deal with problems in the theory of beams, plates and shells. The symplectic technique is all about constructing Hamiltonian formulation from the set of governing equations, that means advancing from Lagrangian system to Hamiltonian system and solutions of which are analyzed in symplectic space. Theory of Hamiltonian system has been introduced long in analytical dynamics. Later mathematicians have noticed that the theory of Hamiltonian is a set of mathematically constructed system [3–5] in which the functions can be separated from their physical meaning. This was turned to be the basic motivation for applying Hamiltonian theory in various fields of mathematics, especially in applied mechanics for theory of beams, plates and shells. In recent times, many researches (see Ref. [6–8]) have been investigated on various approaches to find the exact solution for various problems of beams and plates.

In problems of beams, boundary conditions are given at two ends. In general, conditions given at two ends are combined by constructing new set of algebraic equations. One of the ways of obtaining this set of equations is by applying variational method [9]. In this context we have followed the Hellinger- Reissner variational principle with two kinds of variable.

Classical continuum model be unsuccessful to describe the elastic behavior of the material at high strain-gradient and the dynamic problems of high-frequency vibrations. The reason for this lies in the micro-structure of the material, which is neglected in classical theory. In order to reduce the difference between theoretical results and experimental facts micropolar continuum has been developed. The deformation of a micropolar continuum is described by the position vector to model the translation as it is done in classical model. In addition to that three orthonormal

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vectors, so called rigid directors are introduced, which model the orientation changes of material particles. The development of linear micropolar theory was done by Toupin [10,11], Mindlin and Tiersten [12], Koiter [13], Pal'mov [14], Eringen [15], Iesan [16], etc. For more details the books [17–19] may be found useful.

In this paper we have attempted to find a complete set of solutions for the dynamic problems of high-frequency wave propagating through micropolar beam. We posed the problem in Hamiltonian set-up as it is done in the problems of symplectic elasticity. Here we adopted a new technique to solve Hamiltonian canonical equations which is different from the methodologies that are used in conventional symplectic approach and the technique can reduce the complexity of the problem in micropolar domain. It has been shown that the solutions which are obtained in the problem form an adjoint symplectic orthogonal set. In symplectic elasticity it is discussed that there is a specific physical meaning of symplectic orthogonality, which is equivalent to reciprocal theorem of work [20]. This shows that our findings are consistent with this theoretical result.

2. Basic equations

Based on the fundamental works of Eringen [18], in this section we present some basic relations of the linear micropolar elasticity.

(a) Strain–displacement relation:

$$\varepsilon_{kl} = u_{l,k} + e_{lkm}\phi_m;\ \gamma_{kl} = \phi_{l,k} \text{ where } l,k,m = 1,2,3. \tag{1}$$

here the strain measures are represented by the strain–tensor ε_{kl} and wryness tensor γ_{kl} in which $\vec{u}$, $\vec{\varphi}$ denote the macro-displacement and micro-rotation, respectively. 'Comma' in the subscript denotes the differentiation with respect to space variable and$e_{kml}$is the permutation tensor.

(b) The field equations:

$$t_{kl,k} + \rho(f_l - \ddot{u}_l) = 0\ ;\ m_{kl,k} + e_{lmk}t_{mk} + \rho(\ell_l - J\ddot{\phi}_l) = 0 \tag{2}$$

where t_{kl} is the stress–tensor and $m_{kl}$is the couple stress–tensor. Hereρ denotes the density of the material. f_l, ℓ_l and l_l are the lth components of body-force density and body-couple density respectively. J is the microinertia and 'dot' denotes differentiation with respect to time t.

(c) The constitutive equations:

$$t_{kl} = \lambda\varepsilon_{mm}\delta_{kl} + (\mu+\kappa)\varepsilon_{kl} + \mu\varepsilon_{lk}\ m_{kl} = \alpha\gamma_{mm}\delta_{kl} + \beta\gamma_{kl} + \gamma\gamma_{lk} \tag{3}$$

where λ, μ are Lamé constants, $\kappa, \alpha, \beta, \gamma$ are four additional micropolar material moduli and δ_{kl} is Kronecker delta function.

3. Formulation of the problem

Here we consider a rectangular micropolar beam having x-axis is taken as the axis of the beam with respect to the rectangular orthogonal xyz-coordinate system. The cross sectional area of the beam is taken as A and it is occupies the region: $y_- \le y \le y_+$, $-h \le z \le h$. The beam is vibrating at high frequency ω in a way so that plane parallel to xz-plane is remained plane and parallel. Here we also consider that h is considerably small relative to other dimensions of the beam.

With an analogy of Timoshenko beam theory analysis, in this paper we consider

$$u_1 = -z\theta(x,t),\ \ u_2 = 0,\ \ u_3 = w(x,t) \tag{4}$$

$$\varphi_2 = \phi(x,t),\ \ \varphi_1 = \varphi_3 = 0. \tag{5}$$

With the above consideration, the strain measures can be taken in the following form:

$$\varepsilon_{31} = -\theta - \phi,\ \ \varepsilon_{13} = w'(x,t) + \phi,\ \ \ \varepsilon_{11} = -z\theta'(x,t) \tag{6}$$

where prime represents partial differentiation with respect to x.

By using the Eq. (6), the related constitutive equations are given by,

$$t_{13} = (\mu+\kappa)w'(x,t) - \mu\theta + \kappa\phi \tag{7}$$

$$t_{31} = \mu w'(x,t) - (\mu+\kappa)\theta - \kappa\phi \tag{8}$$

$$m_{12} = \beta\phi'(x,t). \tag{9}$$

Now multiplying the balance of equation of linear momentum along x-axis by z and then integrate over the area of cross-section A we obtain,

$$\int_A \left(\frac{\partial t_{11}}{\partial x} + \frac{\partial t_{31}}{\partial z} + f_1 - \rho\ddot{u}_1\right) z\,dydz = 0$$

$$\text{or } \int_A \frac{\partial t_{11}}{\partial x} z\,dydz + (y_+ - y_-)[zt_{31}]_{-h}^{h} - \int_A t_{31}\,dydz + \int_A f_1 z\,dydz - \rho\int_A z\frac{\partial^2\theta}{\partial t^2} z\,dydz = 0$$

$$\text{or } -\int_A Ez\frac{\partial\theta}{\partial x} z\,dydz + (y_+ - y_-)[zt_{31}]_{-h}^{h} - \int_A t_{31}\,dydz + \int_A f_1 z\,dydz - \rho\int_A z\frac{\partial^2\theta}{\partial t^2} z\,dydz = 0$$

$$\text{or } EI\theta''(x,t) + \int_A t_{31}\,dydz + m - \rho I\ddot{\theta}(x,t) = 0 \tag{10}$$

where $t_{11} = E\varepsilon_{11}$ in which $E = \frac{(2\mu+\kappa)(3\lambda+2\mu+\kappa)}{2\lambda+2\mu+\kappa}$ is called the Young's modulus of the material. Idenotes the moment of inertia of the cross sectional area A of the beam about y-axis and $m = -\int_A f_1 z\,dydz$ is the moment of body-force about y-axis.

Similarly integrating linear moment of momentum along z-axis and moment of momentum along y-axis over the area of cross-section where body couple density is ignored, we get other two beam equations, respectively, as follows:

$$\frac{\partial}{\partial x}\left(\int_A t_{13}\,dydz\right) + \int_A f_3\,dydz - \rho A\ddot{w}(x,t) = 0 \tag{11}$$

$$\frac{\partial}{\partial x}\int_A m_{12}\,dydz + \int_A (t_{31} - t_{13})\,dydz - \rho AJ\ddot{\phi}(x,t) = 0 \tag{12}$$

here we ignore body-couple density $\ell = 0$.

Now we have considered the case where wave of high frequency ω is propagating through the micropolar beam. Therefore we consider the following form of solution:

$$(w,\ \theta,\ \phi) = (W(x), \Theta(x), \Phi(x))\exp(\mathrm{i}\omega t) \tag{13}$$

where $i = \sqrt{-1}$.

Using (13) we obtain from Eqs. (10)–(12):

$$\left[kA(\mu+\kappa)\frac{d^2}{dx^2} + A\rho\omega^2\right]W - kA\mu\frac{d\Theta}{dx} + k\kappa A\frac{d\Phi}{dx} + f = 0 \tag{14}$$

$$\left[EI\frac{d^2}{dx^2} + \rho I\omega^2 - Ak(\mu+\kappa)\right]\Theta + kA\mu\frac{dW}{dx} - k\kappa A\Phi + m = 0 \tag{15}$$

$$\left[\beta kA\frac{d^2}{dx^2} + Ak\rho J\omega^2 - 2Ak\kappa\right]\Phi - Ak\kappa\frac{dW}{dx} - Ak\kappa\Theta = 0 \tag{16}$$

where k is shear-correction and for the beam of rectangular cross-section $k \approx 1.2$; $m = m\exp[-i\omega t]f = \exp[-i\omega t]\left[\int_A f_3\,dydz\right]$.