International Journal of Mechanical Sciences 108-109 (2016) 115–121

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Partial slip incomplete contacts under constant normal load and subject to periodic loading

D.A. Hills [a,*], R.M.N. Fleury [a], D. Dini [b]

[a] *Department of Engineering Science, University of Oxford, Parks Road, Oxford OX1 3PJ, UK*
[b] *Department of Mechanical Engineering, Imperial College London, London SW7 2AZ, UK*

ARTICLE INFO

Article history:
Received 17 December 2015
Accepted 28 January 2016
Available online 3 February 2016

Keywords:
Frictional contact
Edge asymptotes
Fretting

ABSTRACT

We present a general formulation for the stick slip behaviour of incomplete contact under oscillating loading, but with a constant normal load. An asymptotic description of the contact traction *very* close to the contact edges is used. The slip zones present in the steady state with cyclically varying bulk tension and shear force (with an arbitrary phase shift) are found. The range of the variation of the state of stress near both of the contact edges and the respective slip zone sizes are defined in terms of the loading parameters, including the phase angle. The quality of the approximations used by the asymptotic approach and the range of applicability of the method is also analysed in detail in this paper.

1. Introduction

The existence of small areas of slip is the dominant factor in all fretting problems. When present it is almost invariably at the edges of a contact, and it is from these points that cracks often nucleate. We know that there are mainly two forms of contact geometry which may arise in *any* engineering problem: first, there are sharp edged contacts which give rise to stick (usually) at the edges and which have to be handled by wedge asymptotic methods. The other class of contact comprises those which are incomplete (dovetails, firtrees, Hertzian contact) and where, because the contact pressure falls smoothly to zero at the edges, there is invariably some local slip. This paper is concerned exclusively with the latter type, and its function is to demonstrate that the asymptotic forms introduced in earlier articles both (a) fully control the extent of slip and (b) may be used to infer the fretting fatigue strength of problems of a very wide range of geometries. Here, it is assumed that the contact is subject to a constant normal load, so that the size of the contact (and the location of the contact edges) remain fixed.

The first study of a stationary partial slip contact problem was carried out by Cattaneo [1] who analysed the Hertz problem. Apparently unaware of this Mindlin looked at the same problem a little later and extended the solution to look at further properties [2–4]; much of Mindlin's work was on the axi-symmetric form of the contact where the Poisson effect was ignored, but the solution has the additional property that it is possible to infer its absolute tangential (as well as normal) compliance. An important feature of Mindlin and Cattaneo's solutions is that they both assume that the origin of shear tractions along the interface is the exertion of an external shear force and, in most practically arising problems this is augmented by the existence of remotely applied differential surface tensions in the contacting bodies, which was first analysed in [5], for the plane case. Whereas the presence of a remote shear induces slip zones of the same sign the exertion of remote tension induces slip zones of opposite sign. The next major development in solutions came with the formulation of the Ciavarella–Jäger theorem [6,7] which showed that the corrective shear traction in the slip zone is a scaled (and possibly shifted) form of the contact pressure. This development has proved invaluable in solving many partial slip contact problems both with and without tension, but its major drawback is that it cannot be applied where slip zones of opposite sign arise.

The proposal here is to model everything at the contact edge by two asymptotic forms [8,9] and to show how those solutions may be used to infer everything about the local edge slip region. The method is very simple to apply, but the price one pays for the simplicity is that it may only be used when the slip region is a relatively small fraction of the contact half-width; that is, it will not work when the problem approaches the sliding condition. On the other hand, that does not often arise in practice, and the major advantage is that we may infer material properties from a simple laboratory test [10] which are transferable between a wide range of prototypical geometries.

* Corresponding author. Fax: +44 01865 273906.
E-mail addresses: david.hills@eng.ox.ac.uk (D.A. Hills), rodolfo.fleury@eng.ox.ac.uk (R.M.N. Fleury), d.dini@imperial.ac.uk (D. Dini).

http://dx.doi.org/10.1016/j.ijmecsci.2016.01.034
0020-7403/

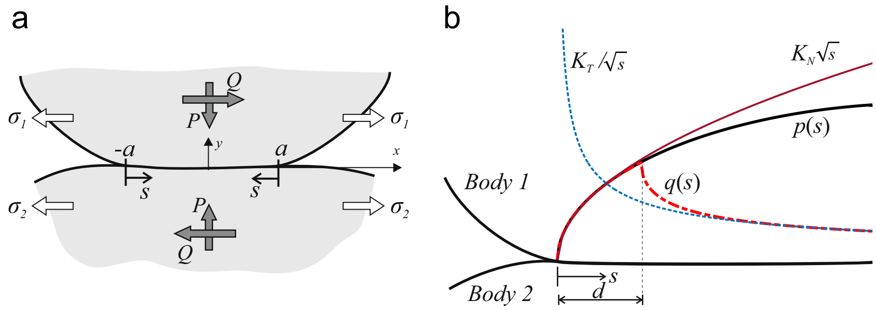

Fig. 1. (a) Incomplete fretting problem under normal, shear and bulk tension loading. (b) Contact tractions and edge asymptotes.

2. Basic asymptotes and their calibration

2.1. Shear contact traction

Consider the incomplete contact shown in schematic form in Fig. 1(a), but note that it need not be Hertzian. We assume, initially, that the coefficient of friction is sufficiently high for all slip to be prevented. If the contact half-width is a, the shearing traction, $q(x)$, induced by a shear force, Q, is given by

$$q(x)=\frac{Q}{\pi\sqrt{a^2-x^2}}. \tag{1}$$

If, on the other hand, remote bulk tensions, σ_1 in the upper component and σ_2 in the lower component are applied in the two bodies, a shear traction distribution of the form

$$q(x)=-\frac{\sigma_0 x}{4\sqrt{a^2-x^2}}, \tag{2}$$

arises, where $\sigma_0=\sigma_2-\sigma_1$. A shift of coordinates to the left hand edge $(x=-a)$ is effected by setting $s=a+x$, and we may write the shear traction in the neighbourhood of this point, $s\to 0$, and neglecting the higher order terms, in the form

$$q(s)=\frac{K_T}{\sqrt{s}}, \tag{3}$$

where

$$K_T=\frac{Q}{\pi\sqrt{2a}}+\frac{\sigma_0}{4}\sqrt{\frac{a}{2}}. \tag{4}$$

It should be emphasized that this result is universal in the sense that it applies to contacts of any geometric form, subject only to the requirement that each body is capable of idealization by a half-plane. In some cases, the half-plane idealisation is applicable *only* near the edge of the contact (e.g. elastically similar flat and rounded punch) but, even if the centre of contact does not behave as a half-plane, the approach can still be implemented [11]. We adopt here a *local* convention that a positive value of K_T implies, at each contact edge in body 2, a shear traction which is directed inwards, away from the contact corner. In body 1, the opposite occurs and K_T is positive when the shear traction acts outwards. Note that when a positive shear force, Q, is applied as in Fig. 1(a), the shear traction in body 2 is directed inwards on the left hand edge, but outwards on the right hand edge. On the other hand, when a positive bulk tension, σ_0, is applied, the shear traction is directed inwards in body 2 on both the left and right hand edges. At the right hand contact edge, we find that

$$K_T=-\frac{Q}{\pi\sqrt{2a}}+\frac{\sigma_0}{4}\sqrt{\frac{a}{2}}. \tag{5}$$

The tractions on the left hand edge act inwards on body 2, i.e. K_T is *positive*, if

$$\frac{\sigma_0 a}{Q}<-\frac{4}{\pi}, \tag{6}$$

and on the right hand edge if

$$\frac{\sigma_0 a}{Q}>\frac{4}{\pi}. \tag{7}$$

2.2. Normal contact pressure

We turn, now, to a description of the contact pressure, $p(s)$, in the neighbourhood of the contact edge. A single term is used and, for half-plane problems, we know from basic Riemann–Hilbert theory that it must decay in a square root bounded manner. Hence, we define the pressure near the contact edge in terms of the normal edge scaling factor, K_N, as

$$p(s)=K_N\sqrt{s}. \tag{8}$$

This *is* geometry dependent and, generally, where the contact problem itself is solved by, for example, the finite element method, we can find the value by plotting $p(s)/\sqrt{s}$, and inferring the value as $s\to 0$. Note that results very close to the contact edge may need to be discarded if convergence of the solution is imperfect. In the case of contacts having a simple geometry we can find the calibration for K_N in closed form, and this is done in Appendix A for a Hertzian contact [8], for contact of shallow wedges [14], and for the cases of slightly rounded contact [11–13].

3. The partial slip problem

3.1. Size of the slip zone in the steady state loading regime

The contact pressure in the slip regions (if small) is encapsulated in the value of K_N, and so too is the magnitude of the shear tractions if the coefficient of friction, f, is specified. What remains to be determined is simply the extent of the slip zone, d. In any cyclic loading problem, unless the applied loading is fully reversing, some frictional shakedown will always occur within the first cycle so that the steady state extent of slip, d, is independent of the mean value of Q or σ_0 or, here, K_T. It is only the *range*, ΔK_T, which matters. So, for any closed loop trajectory in $Q-\sigma_0$ space we need only find how this maps into K_T in order to find the range itself. The steady state slip length (and of course the forwards and backwards slip lengths must be of the same extent, for conservation of material) is given by [9]

$$fd=\frac{\Delta K_T}{K_N}. \tag{9}$$

In fact, the mean value of K_T will affect the solution only in so far as it controls the residual interfacial shearing tractions (or residual interfacial slip displacement distribution) in the stick region. But there is no reason to suppose that this would have any bearing on the fatigue strength of the contact.

Eq. (9) is different from the slip zone size under monotonically increasing loading by a factor 2. The slip zone for the initial loading phase was derived by Dini and Hills [8] and in the presence of an