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Stress analysis of rotating cylindrical shell composed of functionally graded incompressible hyperelastic materials

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A R T I C L E I N F O

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In this paper, rotating thick-walled hollow cylindrical shell composed of functionally graded material are analyzed by using the theory of hyperelasticity. Hyperelastic behavior is modeled by using proposed power law strain energy function with variable material parameters. Material is considered incompressible and material properties are assumed as a function of the radius of the cylinder to a power law function. Material inhomogeneity parameter (n) is a power in the mentioned power law function. Material constant of strain energy function is calculated from experimental data by using Levenberg–Marquardt nonlinear regression method. The analytical solution is obtained for the axisymmetric plane strain state. Following this, profiles of circumferential stretch, radial stress, circumferential stress and longitudinal stress as a function of radial direction are plotted for different values of n. The obtained results show that the material inhomogeneity parameter (n) and structure parameter (β: ratio of outer radius to inner radius) have a significant influence on the mechanical behavior of rotating thick-walled hollow cylindrical shell made of functionally graded materials with power law varying properties. Thus with selecting a proper n and structure parameter (β), engineers can design a specific FGM hollow cylinder that can meet some special requirements.

1. Introduction

In general, rubber-like materials display nonlinear behavior described by hyperelasticity. Hyperelasticity is the ability of a material to incident large elastic strain due to small forces, without losing its original properties. Specific characteristics and economic advantages [1] of rubber like materials caused widely uses of these materials in various structures of different industries such as shells, tubes, rings, spheres and pads, in petrochemical, aerospace, biomedical and many other fields of human life.

Many efforts have been made to expand a theoretical stress–strain relation that fits experimental results for hyperelastic materials. Simple constitutive relations for studying their mechanical behavior include the neo-Hookean and the Mooney–Rivlin strain energy functions. Mooney [2] offered a theory of large elastic deformation, Rivlin considered large elastic deformations of rubber. Blatz–Ko [3] offered a new strain energy function of rubber like materials. Yeoh [4] suggested a strain–energy function for the description behavior of carbon-black-filled rubber vulcanized. Ogden [5] constructed an energy function for description of rubber-like solids for nonlinear large elastic deformations based on strain energy density functions. In each of these methods, a set of coefficients must be determined. Review of mechanical behavior of rubber like materials could be found in the articles contributed by Beatty [6], Horgan and Polignone [7], Attard [8] and the monograph contributed by Fu and Ogden [9]. In recent years several researches were done on constitutive modeling of rubber like materials such as works by Anani and Alizadeh [10], Tomita et al. [11], Coelho et al. [12] and Santos et al. [13].

The main concern of this paper is thick rotating cylindrical shell made of FG rubber like materials, therefore studies and investigations on different axisymmetric shells are carefully reviewed and their key notes are mentioned here. Plane strain and plane stress analytical solutions of thick hollow cylinder problems in the elastic stress state have been presented for many years in typical and complex textbooks for example, by Timoshenko and Goodier [14]. For specific thickness variations closed form solutions have been found and an approximate technique for any thickness variation was given by Haerle [15]. The disk is split into a series of coaxial rings and a graphical method is used to gather these rings to estimate the original disk. In addition, Feng [16] considered large elastic deformation of incompressible rotating disks and applied a numerical integration method to the governing equations for disks of a Mooney material. The governing differential equations are tremendously weighty when compressible materials are considered and the numerical solution of these equations and

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boundary conditions presents significant difficulties. Also disks whose thickness and elastic properties vary in the radial direction can be considered. The study of anisotropic spinning disks which experience large elastic deformation is of significance in the consideration of superflywheels as argued by Rabenhorst [17]. The use of nonlinear governing differential equations is evaded by considering the disk to be a collective of separate thin coaxial rings, equations of motion, force balance and compatibility relations being formulated for the rings. A set of nonlinear algebraic equations is achieved which must be solved rather than the governing differential equation. This is the extension to generalized plane stress of the idea used by Haddow and Faulkner [18] in considering a thick spherical shell.

Graded rubber like materials were created by Ikeda in the laboratory for the first time [19], a while after these materials have attracted the attention of researchers for modeling these materials behavior under mechanical and geometrical boundary conditions. Some important and new researches about stress analysis of inhomogenous rubber like materials structures are mentioned here. For example, Effects of material inhomogeneities on stress distributions through-the-thickness of circular cylinders made of rubber like materials in mechanical and thermal load was studied by Bilgili et al. [20]. In another study, Bilgili [21] investigated plane strain deformations of a circular cylinder made of an inhomogeneous neo-Hookean material with circumferential displacements prescribed on the inner and the outer surfaces. Torsion of a cylinder made of incompressible Hookean material with the shear modulus varying along the axial direction was studied by Batra [22] and axial variation of the shear modulus to manage the angle of twist of a cross-section was found.

Numerical analysis of plane strain axisymmetric deformations of a circular cylinder made of an inhomogeneous Mooney–Rivlin material was studied via Finite element method by Batra [23]. Batra and Bahrami [24] considered cylindrical pressure vessel made of FG rubber like material under internal pressure. To discover stress components of the pressure vessel, they assumed axisymmetric radial deformations of a circular cylinder composed of FG Mooney–Rivlin material with the material parameters varying continuously through the radial direction either by a power law or an affine relation. Recently, Anani and Rahimi [25] studied behavior of spherical shell made of FG rubbers. They assumed that material properties vary in radial direction by power law function and found distribution of stretch and stress components through the shell thickness.

Surveying through the researches on different structures it is found that there is no literature in analyzing rotating cylindrical shell composed of isotropic FG rubber like materials. Therefore, stress analysis of above mentioned structure is the point of this study and exact analytical solution for stress components and displacements in the plane strain condition of rotating thick-walled hollow cylindrical shell made of isotropic FG rubber like materials is derived.

2. Problem formulation

Fig. 1 shows a rotating thick-walled hollow cylindrical shell made of isotropic rubber like materials with an inner radius A and an outer radius B and constant angular speed ω about its axis in the plane strain condition, and P_i and P_o are represented internal and external pressure. The cylinder is considered initially stress-free and assumed to be deformed statically. Undeformed and deformed configurations of cylindrical shell are considered (R,Θ,Z) and (r,θ,z), respectively. In these configurations the geometry of the cylinder is described as follows:

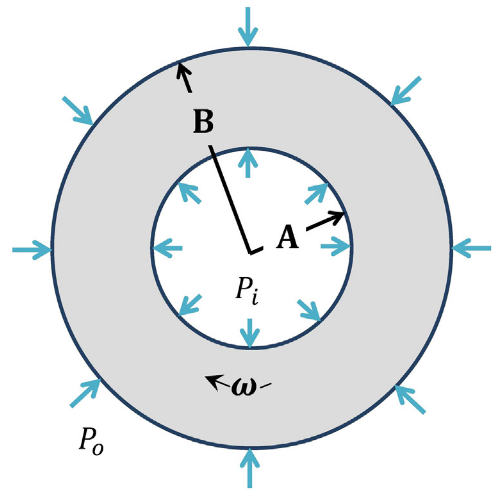

Fig. 1. Configuration of rotating thick cylinder.

$$A \le R \le B, 0 \le \Theta \le 2\pi, 0 \le Z \le L \tag{1}$$

$$a \le r \le b, 0 \le \theta \le 2\pi, 0 \le z \le l \tag{2}$$

Deformation field of the cylinder is member of the family of universal solutions which is proposed by Ericksen [26]:

$$r = r(R), \theta = \Theta, z = Z \tag{3}$$

The deformation gradient $\boldsymbol{F}$, left Cauchy–Green deformation tensor $\boldsymbol{B}$ and principal invariants of the left Cauchy–Green deformation tensor (I_1, I_2, I_3) are given by

$$\boldsymbol{F} = \begin{bmatrix} \frac{dr}{dR} & 0 & 0 \\ 0 & \frac{r}{R} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

$$\boldsymbol{B} = \boldsymbol{F}\boldsymbol{F}^{\boldsymbol{T}} = \begin{bmatrix} \left(\frac{dr}{dR}\right)^2 & 0 & 0 \\ 0 & \left(\frac{r}{R}\right)^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5}$$

$$I_1 = \left(\frac{dr}{dR}\right)^2 + \left(\frac{r}{R}\right)^2 + 1, I_2 = \left(\frac{dr}{dR}\right)^2 \left(\frac{r}{R}\right)^2 + \left(\frac{r}{R}\right)^2 + \left(\frac{dr}{dR}\right)^2, I_3 = J^2 = 1 \tag{6}$$

where J is determinant of the deformation gradient $\boldsymbol{F}$, Incompressibility condition implies: $I_3 = J^2 = 1$, therefore $(r dr)/(R dR) = 1$, and radial deformation is considered:

$$R^2 = r^2 - (b^2 - B^2) \tag{7}$$

where b is outer radius of the cylinder in deformed configuration. Boundary condition has been used to find b. Following parameters are introduced for simplicity:

$$\beta = \frac{B}{A}, \lambda_{(\theta)b} = \frac{b}{B}, \frac{a}{A} = \left(\beta^2\left(\lambda_{(\theta)b}{}^2 - 1\right) + 1\right)^{\frac{1}{2}} \tag{8}$$

where $\lambda_{(\theta)b}$ is hoop stretch in the outer radius and a is inner radius in the deformed configuration. For incompressible hyperelastic materials, Cauchy stress is

$$\boldsymbol{\sigma} = -p\boldsymbol{I} + 2W_1\boldsymbol{B} + 2W_2(I_1\boldsymbol{B} - \boldsymbol{B}^2) \tag{9}$$

where p is the hydrostatic pressure connecting to the incompressibility constraint. W is strain energy function and W_1 and W_2 are derivatives of W respect to I_1 and I_2 respectively and $\boldsymbol{I}$ is identity matrix. The equilibrium equation of the thick-walled rotating cylinder in the radial direction and boundary conditions