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Frictional receding contact analysis of a layer on a half-plane subjected to semi-infinite surface pressure

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A B S T R A C T

Though receding contacts occur in many engineering components, such as bolted joints, their properties are little understood. Previous studies have shown that the contact area in receding contacts reduces with the application of load and is load independent. Here an infinitely long layer in contact with a half-plane of the same material and subject to semi-infinite uniform normal surface pressure applied to the surface of the layer everywhere except along a portion of finite width is solved by applying distributions of edge dislocations. Solutions for the location of the areas of slip and separation along the contact interface are obtained for a variety of coefficient of friction values and no-pressure zone widths. The contact tractions are also computed.

1. Introduction

While receding contacts are present in many mechanical components and structures, including bolted joints, their physical properties are not understood well, and the literature regarding them is sparse. A detailed understanding of the physical properties of receding contacts is required to model accurately the damping properties, fretting damage and subsequent failure of many mechanical and structural components that involve receding contacts.

Keer et al. conducted some work on receding contacts in the 1970s [1,2]. Most recent studies, such as by El-Borgi et al., have investigated receding contacts, under different loading conditions and for different material properties, assuming frictionless contact surfaces [3–7]. Further, two recent studies that included frictional effects assumed slip throughout the contact area during a fully sliding situation [8,9]. There is however a gap in the literature regarding the fundamental quasi-static frictional behaviour of these class of contacts when stick, slip and separation are present.

Ahn and Barber solved using finite elements a quasi-static receding contact problem under-cyclic loading taking into account both stick and slip [10]. Further, Chaise et al. solved a 2D small-strain linear elastic contact problem of a semi-infinite layer on a half-plane of the same material subject to a line load, also taking into account of both stick and slip in the contact region [11]. In this paper, the analysis is extended and a contact problem with a semi-infinite layer, of thickness c, in contact with a half-plane, $y \leq 0$, subject to surface pressure, as shown in Fig. 1, is solved. Both the layer and half-plane are of the same homogeneous isotropic material.

Normal pressure, $\sigma_{yy}(x, c)$, is applied to the points (x,c) on the surface of the layer, $y = c$, everywhere except over a region of width $2a$, such that

$$\sigma_{yy}(x, c) = -p, \quad x \geq a,$$
$$= 0, \quad x < a.$$

The origin is taken to be at the contact interface, equidistant from the two pressure patches. Though the problem loading and geometry is not immediately applicable to an analysis of a specific practical situation, such as in a bolted joint, the solution to this problem should provide insights into the fundamental properties of receding contacts under pressure loading. This problem is particularly attractive since only two parameters are involved in the problem definition—a/c, which defines the geometry and the loading of the problem, and the coefficient of friction, f. Further the difficulties due to lift-off of the layer at infinite distances away from the contact region are avoided.

In receding contact problems such as the problem studied here, the separation of surfaces is instantaneous and discontinuous with the application of any finite load, making modelling with finite elements cumbersome due to the discontinuous separation of a large number of nodes. In this paper, as was done by Chaise et al. [11], a more rigorous solution is obtained by the application of distributed dislocations and solving numerically the integral equations that result to ensure contact conditions are satisfied.

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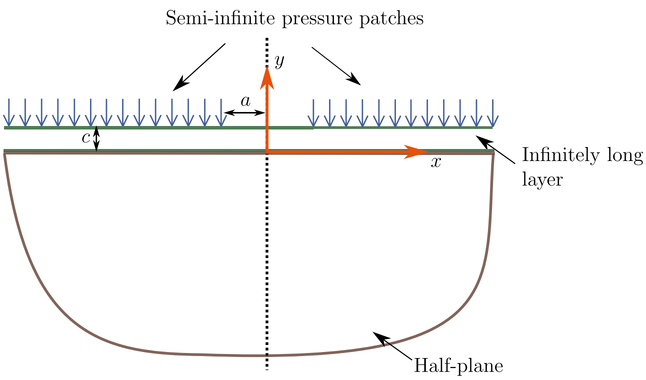

Fig. 1. The problem geometry and loading.

Unlike the problem solved by Chaise et al. separation of points at infinite distances from the contact region does not occur and hence additional complexities and approximations required in the solution of the integral equations are avoided.

2. Solution using distributed dislocations

In this solution method, the layer and the half-plane in contact are modelled as though they are fused as one body making a half plane, $y \le c$, and a distribution of climb and glide dislocations are applied along the surface $y=0$, at the location of the contact surface in the problem definition, to simulate the effect of the contact. These dislocations are not lattice defects but represent displacement discontinuities. For a detailed exposition of the technique, refer to Hills et al. [12].

The bilateral stress components are the stress components due to the external loading on the half-plane, $y \le c$. The total stress components for any point on the layer or the half-plane are given by the sum of the bilateral stress components and the stress components at the point due to the presence of the distributions of dislocations.

Coloumb's friction law states that at a point $(x, 0)$ along the contact interface

$$|\sigma_{xy}(x,0)| \le f|\sigma_{yy}(x,0)|,$$

where $\sigma_{xy}(x,0)$ and $\sigma_{yy}(x,0)$ are the shear and the normal traction components that act at the point $(x,0)$ on the contact surface respectively. Slip occurs when $\sigma_{xy}(x,0) = f\sigma_{yy}(x,0)$. Further, the Signorini condition stipulates that when separation occurs at a point $(x,0)$

$$\sigma_{yy}(x,0) = 0,$$

and when separation does not occur

$$\sigma_{yy}(x,0) < 0.$$

Stick occurs when there is neither separation nor slip, i.e. $\sigma_{yy}(x,0) < 0$ and $|\sigma_{xy}(x,0)| < f|\sigma_{yy}(x,0)|$.

For values of f and a/c for which the tractions determined by the bilateral solution are found not to correspond to stick throughout the contact surface, distributions of glide and if required climb dislocations are applied to regions along the $y=0$ surface. The location of the distributions is computed so that the regions containing climb and glide dislocations satisfy the conditions for slip and separation respectively, and the condition for stick is satisfied at all other points at the contact surface.

The bilateral solution for the traction components along the contact interface, $y=0$, is found by the appropriate integration of Flamant's solution for a normal line force on the half plane, $y \le c$, over the area to which the pressure is applied in this problem.

The bilateral normal and shear traction components, $\sigma_{yyb}(x,0)$ and $\sigma_{xyb}(x,0)$, that act at points $(x,0)$ along the contact surface were thus found to be given by

$$\frac{\sigma_{yyb}(x,0)}{p} = \frac{1}{\pi\left((a+x)^2+c^2\right)}\left[(a+x)c - \left((a+x)^2+c^2\right)\text{arc tan}\left(\frac{c}{a+x}\right)\right] + \frac{1}{\pi\left((a-x)^2+c^2\right)}\left[(a-x)c - \left((a-x)^2+c^2\right)\text{arc tan}\left(\frac{c}{a-x}\right)\right], \quad (1)$$

$$\frac{\sigma_{xyb}(x,0)}{p} = \frac{c^2}{\pi\left((a+x)^2+c^2\right)} - \frac{c^2}{\pi\left((a-x)^2+c^2\right)}. \quad (2)$$

It is expected that the normal traction will be symmetrical and the shear traction will be anti-symmetrical with respect to the y-axis. Depending on the coefficient of friction and the a/c value, three possible outcomes are envisioned.

1. Stick occurs everywhere along the contact interface: in this case the bilateral solution wholly satisfies the condition for stick at every point along the contact interface.
2. Slip occurs along portions of the contact interface, but separation does not occur anywhere at the interface: it is expected that there will be two regions of slip, two on either side of the y-axis, with a stick zone at the centre, see Fig. 2. In modelling this case, a distribution of glide dislocations is applied to portions of the contact area to satisfy the conditions for slip and stick along the interface.
3. Both slip and separation occur along portions of the contact interface: it is expected that separation and stick zones will

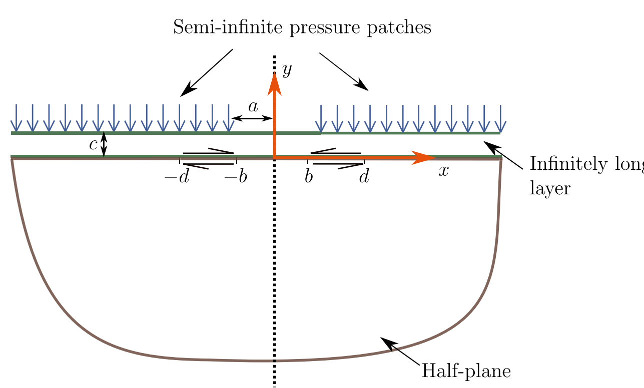

Fig. 2. Solution in which slip but no separation occurs. Slip is expected to occur in two regions along the contact interface.

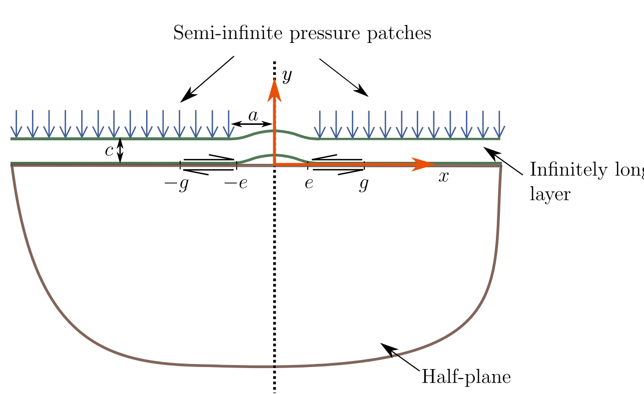

Fig. 3. Solution in which both slip and separation occur. Separation is expected to occur at the central region of the contact interface along the interval $[-e \;\; e]$ and slip is expected to occur along the contact interface along the interval $[-g \;\; g]$.