International Journal of Mechanical Sciences 100 (2015) 112–125

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Analysis of flexural wave bandgaps in periodic plate structures using differential quadrature element method

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ARTICLE INFO

Article history:
Received 11 March 2015
Received in revised form
16 June 2015
Accepted 18 June 2015
Available online 26 June 2015

Keywords:
Periodic composite plate
Differential quadrature element method
Frequency band gap

ABSTRACT

By employing the first order shear deformation plate theory and the Bloch–Floquet theorem, the dispersion equation of flexural wave in the periodic composite plate structure with piezoelectric patches is derived and solved by the use of the differential quadrature element method. Moreover, wave modes for the dispersion curves of the considered periodic plate are compared with those of a homogeneous plate, from which the reason of the frequency band gap is revealed. Then, a comprehensive parametric study is conducted to highlight the influences of the physical parameters and the geometrical parameters on the frequency band gaps. The results show that the method is efficient and accurate and the bandwidth can be enlarged by changing the physical and geometrical parameters. The special band gap property of periodic plate structure has many potential applications in wave/vibrations attenuation areas for mechanical, aerospace and civil engineering structures.

1. Introduction

Periodic composite materials or structures are widely used in many engineering fields. Over the past several decades, the static and dynamic performances of periodic composite plate structures have been extensively investigated [1,2]. By applying the tolerance averaging method developed by Woźniak and Wierzbicki [3], Jędrysiak [4,5] investigated the dynamic behavior of periodic plates, in which the effect of the periodicity cell size on the overall plate behavior is analyzed in the free vibration problem. Using the concept of a representative volume element (RVE) and applying appropriate periodic boundary conditions, Würkner et al. [6,7] developed a numerical homogenization technique to derive the effective material properties of unidirectional periodic fiber composite with imperfect interface conditions between the reinforcement and the filler. By combining the recently proposed finite-volume direct averaging micromechanics (FVDAM) theory [8] with the Particle Swarm Optimization (PSO) algorithm, Tu and Pindera [9] presented a new homogenization-based computational technology for the identification of optimal bio-inspired material architectures (periodic bio-material) that mimic the mechanical response of a biological tissues with wavy microstructures. Using a modified version of the asymptotic expansion homogenization method, Chatzigeorgiou et al. [10] and Tsalis et al. [11] studied the effective thermomechanical properties of composites with cylindrical periodicity and generalized periodicity in the microstructures. Based on multi-node elements whose shape functions are computed numerically by means of an auxiliary fine scale discretization of the element itself, Casadei [12] and Casadei et al. [13] presented a geometric multi-scale finite element method (GMsFEM) for predicting the static and dynamic response of heterogeneous materials and structures. Using the asymptotic homogenization techniques, Andrianov et al. [14] provided a detailed investigation on the analysis of viscoelastic-matrix fibrous composites with square-lattice reinforcement and their effective properties.

In 1993, a special kind of periodic composite structure, named *phononic crystal*, is proposed for the first time by Kushwaha et al. [15] in the solid-state physics. These hetero structures exhibit a fascinating property, called frequency band gap, in which waves or vibrations are forbidden by the periodic composite structure [16–18]. The physical meaning and possible applications of such dispersion property have got considerable attention by a large number of scientists and engineers [19,20]. For example, Andrianov et al. [21] presented an detail application of the higher order asymptotic homogenization method (AHM) to the study of wave dispersion in periodic composite materials, from which it is found that successive reflections and refractions of the waves at the component interfaces lead to the formation of a complicated

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http://dx.doi.org/10.1016/j.ijmecsci.2015.06.014
0020-7403/

sequence of the pass and stop frequency bands when the wavelength of a traveling signal becomes comparable with the size of heterogeneities. By using the AHM, Andrianov et al. [22,23] also studied propagation of strain waves in nonlinear elastic media with microstructure. In the work, the geometrical nonlinearity is described by the Cauchy–Green strain tensor and the physical nonlinearity is described by elastic potential. In the case of weak nonlinearity, an asymptotic solution is developed and a number of nonlinear phenomena are detected, such as generation of high-order modes and localization. Bacigalupo and Gambarotta [24,25] proposed a second-order computational homogenization method to derive the overall constitutive moduli and inertia properties, and evaluated the influences of the material characteristic lengths on the structural response and the propagation of elastic waves in periodic masonry structure. Ruzzene and his co-authors [26,27] developed a perturbation approach for analyzing dispersion and group velocity in nonlinear periodic structures. Unlike other perturbation techniques commonly used to study continuous nonlinear systems, the proposed approach is valid for both large and small wavelengths in the discrete setting. And by using the perturbation approach, the amplitude-dependent band gaps and wave directivity in the anisotropic setting are identified for two-dimensional weakly nonlinear periodic structures. Based on the first-order shear deformable plate theory, Hsu and Wu [28] developed an efficient formulation to calculate and discuss the characteristics of Lamb waves in periodic composite plates. Further, the method is applied to study the propagation of Lamb waves in locally resonant periodic composite plates [29]. With the help of the center finite difference method (CFDM), Zhou et al. [30] analyzed the frequency band gaps of a periodically stiffened thin plate (PSTP) structure. Later on, they developed a simplified super element method (SSEM) to investigate the effects of the material damping on the flexural vibration characteristic of the periodically stiffened-thin-plate [31]. Additionally, various ways for adjusting the frequency band gap were already described in the literature for periodic composite structures [32,33]. The application of smart materials for adjusting the frequency band gaps of periodic plate structures has also received great attention [34,35].

In order to analyze the frequency band gap property of periodic composite structure reasonably, various numerical and theoretical methods have been successfully developed, such as the widely-used plane wave expansion method (PWE), the multiple scattering theory method (MST), the finite difference time domain method (FDTD), the transfer matrix method (TM) and so on. However, the PWE method encounters convergence problems when the periodic material has a large elastic mismatch [36]; the MST method has a limitation in studying periodic materials with overlap scatters [37]; the FDTD method is very memory and CPU-time intensive to satisfy the stability condition [38]; the TM method cannot be used to calculate the dispersion curves of two-dimensional and three-dimensional periodic plate structure. Therefore, alternative computational methods are always expected. Differential quadrature method (DQM) has been tested to be an efficient numerical method for solving both linear and nonlinear partial differential equations, and it has the potential to become an alternative to the conventional numerical methods [39,40]. In 2009, Xiang and Shi [37] extended the DQEM to the analysis of flexural vibration frequency band gaps of periodic beams and demonstrated that the method is very accurate and simple. Furthermore, they developed the DQEM to analyze the band structures and dynamic response of a periodic Timoshenko beam resting on a Pasternak foundation [41]. However, the DQEM has not been used to study the dispersion structures of two-dimensional and three-dimensional periodic structures. Therefore, the main purpose of the present work is to develop the DQEM to calculate the dispersion curves of flexural wave in two-dimensional periodic plate structures and to discuss the frequency band gap property of the periodic plate structure with piezoelectric patches.

The paper is organized as follows. In Section 2, the governing equations for the flexural wave propagating in the composite periodic plate with piezoelectric patches is developed by using

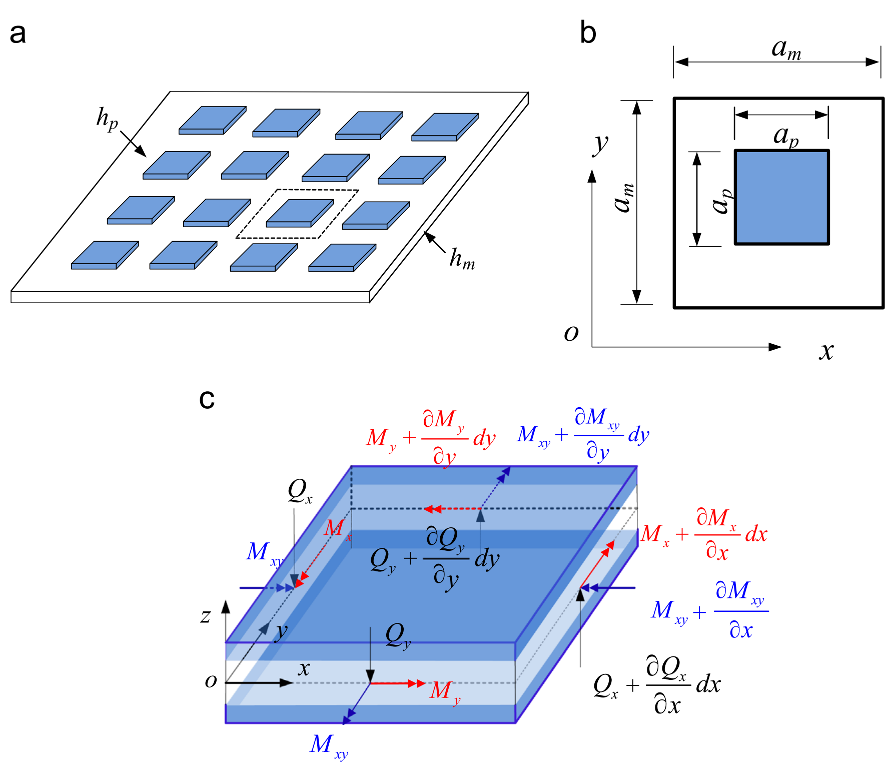

Fig. 1. Illustration of (a) an infinite periodic plate with piezoelectric patches, (b) a typical unit cell, and (c) the bending moments and shear forces of the plate.