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# Buckling behaviour of laminated composite skew plates with various boundary conditions subjected to linearly varying in-plane edge loading

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ABSTRACT

In the present study, the buckling behaviour of laminated composite skew plates with different boundary conditions subjected to linearly varying in-plane loads are presented. The skew plate is modelled based on higher order shear deformation theory, which accurately predicts the buckling behaviour for the thick plate. The in-plane stress distribution within the skew plate due to linearly varying in-plane load is equal to the applied in-plane edge load in the pre-buckling range. Using these in-plane stress distributions, the total potential energy functional is formulated. Total potential energy is a function of the total strain energy of skew plate and potential energy due to in-plane stress distributions. The total strain energy of skew plate contains membrane energy, bending energy, additional bending energy due to additional change in curvature and shear energy due to shear deformation, respectively. The total potential energy functionals mapped from physical domain to computational domain over which a set of orthonormal polynomials satisfying the essential boundary conditions is generated by Gram–Schmidt orthogonalization process. Using a Rayleigh-Ritz method in conjunction with Boundary Characteristics Orthonormal Polynomials, the total potential energy functional is converted into sets of algebraic equations. Finally, these algebraic equations are rearranged as a linear eigenvalue problem, which is solved to obtain the critical buckling loads. The numerical results are presented for different skew angles, boundary conditions, length to thickness ratios, aspect ratios and in-plane loadings. It is observed that the critical buckling load increase with the increase of skew angle as well as change in the mode shape at a lower aspect ratio with the increase of skew angle.


## 1. Introduction

Often, plates are a part of complex structural system and hence load coming on it may not be always uniform. For example, in the case of I-beam beam subjected to bending moment at the ends, the web of the beam is subjected to non-uniform in-plane loads. The load exerted on the aircraft wings, or on the stiffened plate in the ship structures or on the slabs of a multi-storey building by the adjoining structures usually is non-uniform. The type of distribution in an actual structure depends on the relative stiffness of the adjoining element. Behaviour of structures subjected to non-uniform in-plane compressive loading and shear loading is important in aircraft, civil and ship-building industries. Much work has been reported in the literature on the buckling of rectangular plates subjected to uniform in-plane loading. However, few researchers have analysed the static stability analysis of rectangular plates subjected to non-uniform in-plane loading.

Leissa and Kang [1] and Kang and Leissa [2,3] presented exact solutions for the Kirchhoff plate having two opposite edges simply supported subjected to linearly varying in-plane loading. They have considered all other possible boundary conditions on the unloaded edges. As the loaded edge is simply supported, authors assumed the transverse displacement ($w$) to vary as $\sin(m\pi x/a)$ (where '$a$' is the size of the plate along $x$-direction and '$b$' along $y$-direction) and reduced the governing partial differential equation to an ordinary differential equation in $y$ with variable coefficients, for which an exact solution was obtained in terms of power series (i.e., method of Frobenius). Applying the boundary conditions at $y=0$ and $b$ yields the eigenvalue problem for finding the buckling load. Zhong and Gu [4,5] developed the exact solution of the buckling of simply supported symmetrical cross-ply moderately thick laminated plates under a linearly varying edge load. The authors obtained the buckling load for various aspect ratios of the

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composite plates on the basic of first-order shear deformation theory. Wang et al. [6] used a DQ method to study the vibration and buckling of an SS–C–SS–C rectangular plate loaded by linearly varying in-plane stresses. Convergence study showed that the DQ method yields very accurate results as of Leissa and Kang [2]. Lai and Xiang [7] presented the discrete singular convolution (DSC) method for solving buckling and vibration problems of rectangular plates with elastically restrained edges along with linearly varying uni-axial in-plane loading. Authors presented accurate buckling and vibration solutions of plates having two opposite edges elastically restrained and the other two sides clamped. Lopatin and Morozov [8] presented the analytical solution of the buckling problem for an orthotropic rectangular plate having two parallel edges simply supported, one edge clamped and the remaining edge free (the SSCF plate). The plate considered is subjected to a linearly varying in-plane load and the solution technique involves Kantorovich procedure in conjunction with a generalised Galerkin method. The buckling problems are solved for isotropic and orthotropic plates with various aspect ratios. Farajpour and Shahidi [9] investigated the buckling response of orthotropic single layered graphene sheet (SLGS) subjected to linearly varying normal stress using the nonlocal elasticity theory. Differential quadrature method (DQM) is used to solve the governing equations for various boundary conditions.

There are many open literature available on buckling and vibration analysis of skew plate subjected to uniform in-plane loading. Durvasula [10] investigated the buckling problems of simply supported skew plates using Rayleigh-Ritz method, employing fourierseries in oblique coordinates. Liew and Lam [11] studied the flexural vibration of skew plates by the Rayleigh-Ritz method and followed Gram–Schmidt orthogonalization process to generate two-dimensional orthogonal plate functions. Wang et al. [12] analysed buckling of skew plates and corner conditions for simply supported edges by using the Rayleigh-Ritz method. Flexural vibration of skew plates using boundary characteristic orthogonal polynomials in two variables is investigated by Singh and Chakraverty [13]. Wang [14] used B-spline function in conjunction with the Rayleigh-Ritz method for buckling analysis of skew composite laminated plates using first order shear deformation theory.Mizusawa et al. [15] presented a general procedure for calculating the buckling of stiffened skew plates by the Rayleigh-Ritz method with B-spline functions as co-ordinate functions. Taj and Chakrabarti [16] investigated the buckling analysis of functionally graded skew plates within the frame work of a finite element method based on third order shear deformation theory.Kitipornchai et al. [17] determined the buckling solutions for skew plates of various aspect ratios, skew angles and boundary conditions based on the principle of stationary total potential energy in conjunction with a pb-2 Rayleigh-Ritz method. Farag and Ashour [18] developed a fast converging semi-analytical method for assessing the vibration effect on thin orthotropic skew plates. The authors used a Kantorovich method for developing a new modification of the finite strip method for reducing the complexity of the problem. Ganapathi and Prakash [19] investigated the thermal buckling of a simply supported functionally graded skew plate using first-order shear deformation theory in conjunction with the finite element approach. Linear and nonlinear temperature rise across the thickness are considered. The effects of aspect and thickness ratios, gradient index and skew angle on the critical buckling temperature difference are studied.Wang et al. [20] proposed a new version differential quadrature method (DQM) to obtain buckling loads of thin anisotropic rectangular and isotropic skew plates. Numerical results indicate that faster convergence is achieved and excellent results are obtained than the old version DQM. Daripa and Singha [21] investigated stability characteristics of composite skew plates subjected to in-plane compressive load using the shear deformable finite element approach. The influences of high prebuckling stresses at the corner regions of isotropic and composite skew plates on their stability characteristics are emphasised for different load direction, boundary condition and laminate stacking sequence.Karami et al. [22] used the differential quadrature method for static, free vibration, and stability analysis of skewed and trapezoidal composite thin plates. The authors used general transformation scheme for transferring the variation of the variables in the computational to the physical domain and vice versa. Zhou and Zheng [23] employed an MLS-Ritz method for eliminating the numerical difficulties in analysing the vibration of a skew plate with a large skew angle. This method utilises the moving least square technique to establish the trial function for the transverse displacement of a skew plate and the Ritz method is applied to derive the governing eigenvalue equation for the skew plate. The boundary conditions of the plate are enforced through a point substitution technique that forces the MLS-Ritz trial function satisfying the essential boundary conditions along the plate edges. Singha and Ganapathi [24] analysed the large amplitude free flexural vibration of thin laminated composite skew plates using finite element approach. The geometric non-linearity based on von Karman's assumptions is introduced and the governing equations obtained employing Lagrange's equations of motion are solved using the direct iteration technique. Taj and Chakrabarti [16] studied the static and dynamic analysis of functionally graded material (FGM) skew plates under mechanical loading within the frame work of the finite element method. The authors used FEM formulation based on a third-order shear deformation (TOSD) theory that does not require any shear correction factor. Eftekhari and Jafari [25] proposed modified mixed Ritz-differential quadrature (DQ) methodology for free and forced vibration, and buckling analysis of rectangular plates. The modified Ritz method in conjunction with the DQ method is a versatile, accurate and efficient method for free vibration analysis of thick rectangular and skew plate. Wang et al. [26] accurately analysed skew plates with different boundary conditions by using the new version of the differential quadrature method (DQM). The authors verified the accuracy with a conventional finite element method with very fine mesh.

So far, to the best of authors' knowledge, there is no work available in open literature on the buckling behaviour of composite skew plates based on higher order shear deformation theory (HSDT) under linearly varying in-plane loading. In the present investigation, the buckling behaviour of composite skew plates subjected to linearly varying inplane loading is considered for ten sets of boundary conditions. The inplane stress distribution within the skew plate due to linearly varying load is equal to the applied in-plane load in the prebuckling range. Using these in-plane stress distributions, the total potential energy functional is formulated. The total potential energy functional is transformed from physical domain to computational domain using transformation relations. The orthonormal polynomials are generated by using Gram–Schmidt orthogonalization process, which satisfy the essential boundary conditions of skew plate in computational domain. Using the Rayleigh-Ritz method in conjunction with Boundary Characteristics Orthonormal Polynomials (BCOPs), the total potential energy functional is converted into sets of algebraic equations. BCOPs functions consist of the product of two dimensional linearly independent set of polynomial functions and a basis function. The basis function is formed from taking the product of the equations of the boundaries. To satisfying the essential boundary condition, each equation of the boundary is raised to the power of 0, 1 or 2 corresponding to free, simply supported, or clamped edges. The critical buckling load is obtained from the solution of the associated linear eigenvalue problem. The effects of skew angles, boundary conditions, shear deformations, aspect ratios and loadings on the buckling behaviour of composite skew plate are presented.

## 2. Mathematical formulation

Laminated skew plate of length '$a$', width '$b$' having '$l$' layers of equal thickness and the $z$-axis in the direction perpendicular to