International Journal of Mechanical Sciences 100 (2015) 169–179

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Evaluation of the accuracy of classical beam FE models via locking-free hierarchically refined elements

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ARTICLE INFO

Article history:
Received 11 April 2015
Received in revised form
12 June 2015
Accepted 30 June 2015
Available online 8 July 2015

Keywords:
Refined beam theories
6-DOF beam element
Finite elements
MITC
Carrera unified formulation

ABSTRACT

It is well known that the classical 6-DOF (Degrees of Freedom) beam theories that are incorporated in commercial finite element (FE) tools are not able to foresee higher-order phenomena, such as elastic bending/shear coupling, restrained torsional warping and three-dimensional strain effects. In this work, the accuracy of one-dimensional (1D) finite elements based on the classical theories (Euler–Bernoulli and Timoshenko theories as well as a 6-DOF model including torsion) is evaluated for a number of problems of practical interest and modelling guidelines are given. The investigation is carried out by exploiting a novel hierarchical, locking-free, finite beam element based on the well-known Carrera Unified Formulation (CUF). Thanks to CUF, the FE arrays of the novel beam element are written in terms of fundamental nuclei, which are invariant with respect to the theory approximation order. Thus, results from classical as well as arbitrarily refined beam models can be formally obtained by the same CUF beam element. Linear Lagrange shape functions are used in this paper to interpolate the generalized unknowns and shear locking phenomena are avoided by adopting a MITC (Mixed Interpolation of Tensorial Components) scheme. Different sample problems are addressed, including rectangular and warping-free circular cross-sections as well as thin-walled beams. The results from classical theories and the 6-DOF model are compared to those from higher-order refined beam models, both in terms of displacement and stress fields for various loading conditions. The discussion focuses on the limitations of the commonly used 1D FEs and the need for refined kinematics beams for most of the problems of common interest. The research clearly depicts CUF as a valuable framework to assess FE formulations such as the 6-DOF model herein considered, which is one of the most known and used finite element for the analysis of structures.

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1. Introduction

One of the reasons for the success of the Finite Element Method (FEM) in solid mechanics is due to the use of 6-DOF (Degrees of Freedom) models into commercial tools. This choice, formerly adopted by Nastran codes, allows, in fact, the analysts to deal with only physical unknown quantities (e.g., translations and rotations in displacement-based formulations for pure mechanical problems). Moreover, mathematical models of complex structures can be straightforwardly constructed by assembling finite elements of different type and orientation, see for example the reinforced shell-like structures for aerospace applications [1]. Nevertheless, it is clear that limiting the maximum number of DOF per node can introduce certain physical inconsistencies. For this reason, most of the commercially available FEM software tools makes use of fictitious corrections, such as shear and warping correction factors (see for example [2]). Over the years, many scientists have been working on improved theories to overcome the limitations of classical models. However, their research, besides a few cases, rarely influenced the development of the commercial tools because of the aforementioned limitation of the 6-DOF per node. For the sake of completeness, a brief and not comprehensive review of higher-order theories is provided in the following. The attention is mainly focussed on beam modelling, which represents the principal subject of the proposed work.

Several examples of refined beam models can be found in well-known books on the theory of elasticity, for example, the book by Novozhilov [3]. A possible grouping of the methodologies developed to build higher-order beam theories could be the following: (i) the use of warping functions; (ii) the Saint-Venant based 3D solutions and the Proper Generalized Decomposition method (PGD); (iii) the Variational Asymptotic Method (VAM); (iv) and the Generalized Beam Theory (GBT). The introduction of warping functions to improve the displacement field of beams is a well-known strategy. Warping functions were first introduced in the

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http://dx.doi.org/10.1016/j.ijmecsci.2015.06.021
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framework of the Saint-Venant torsion problem [4–6]. Some of the earliest contributions to this approach were those by Umanskij [7], Vlasov [8] and Benscoter [9]. The Saint-Venant solution has been the theoretical basis of many advanced beam models. Three-dimensional elasticity equations were reduced to beam-like structures by Ladevèze and his co-workers [10]. Using this approach, a beam model can be built as the sum of a Saint-Venant part and a residual part and then applied to thick beams and thin-walled sections. The PGD for structural mechanics was first introduced in [11]. The PGD can be considered as a powerful tool to reduce the numerical complexity of a 3D problem. Bognet et al. [12] applied PGD to plate/shell problems, whereas Vidal et al. [13] extended PGD to beams. Asymptotic methods represent a powerful tool to develop structural models. In the beam model scenario, the works by Berdichevsky [14] and Berdichevsky et al. [15] were among the earliest contributions that exploited the VAM. These works introduced an alternative approach to constructing refined beam theories in which a characteristic parameter (e.g., the cross-section thickness of a beam) is exploited to build an asymptotic series. Those terms that exhibit the same order of magnitude as the parameter when it vanishes are retained. Some valuable contributions on asymptotic methods are those related to VABS models, as in Volovoi et al. [16]. The GBT has been derived from Schardts work [17,18]. The GBT enhances classical theories by exploiting a piece-wise description of thin-walled sections. It has been employed extensively and extended, in various forms, by Silvestre and Camotim, and their co-workers (see for example [19]). Many other higher-order theories, based on enhanced displacement fields over the beam cross-section, have been introduced to include non-classical effects. Some considerations on higher-order beam theories were made by Washizu [20]. Other refined beam models can be found in the excellent review by Kapania and Raciti [21,22], which focused on bending, vibration, wave propagations, buckling and post-buckling. For further details about beam models, the reader is also referred to [23].

Most of the refined theory in the literature are problem dependent. Conversely, according to the well-known Carrera Unified Formulation (CUF), higher-order kinematics can be hierarchically developed in an automatic manner (see [24]).

Regarding beam theories, CUF has been successfully applied to thin-walled structures [25,26], buckling problems [27], free vibration and dynamic response analyses [28,29], composite structures [30,31] and component-wise analysis of aerospace and civil structures [32,33].

The principal characteristic of CUF models is that the order of the theory is a free parameter, or an input, of the analysis. Hence, in a FEM framework, classical and arbitrarily refined elements can be formally developed by using the same formulation. This makes CUF a valuable tool to evaluate the accuracy of any structural model in a unified manner, see for example [32,34].

This property of CUF is therefore exploited in the present paper, whose aim is to assess the accuracy of classical finite beam elements, such as those based on the Euler–Bernoulli Beam Model (EBBM), the Timoshenko Beam Model (TBM), and the 6-DOF beam model including twisting.

In fact, a two-node, locking-free, CUF finite beam element is developed in the following and used to obtain classical and refined results of compact and thin-walled cross-section beam structures undergoing various loading conditions.

Shear locking phenomena are overcome in this work by adopting an MITC (Mixed Interpolation of Tensorial Components) technique, see [35–37]. The MITC formulation allows the transverse shear locking phenomenon to be eliminated by introducing an independent finite element approximation into the element domains for the transverse shear strains.

This work is organized as follows: (i) first, classical beam theories are formulated in the framework of CUF; (ii) higher-order models are then developed by approximating the beam kinematics via arbitrarily truncated expansion series; (iii) next, an MITC finite element formulation is outlined in Section 4; (iv) subsequently, the novel beam element is used to analyse various problems and the results of classical beam elements are compared to those from higher-order models; (v) finally, some comments and guidelines are discussed.

2. Classical beam theories

Fig. 1 shows a generic beam and the Cartesian coordinate system adopted. The beam is depicted with a rectangular cross-section. However, this choice does not affect the validity of the proposed formulation.

The kinematic field of the Euler–Bernoulli Beam Model (EBBM) can be written as

$$\begin{aligned} u_x &= u_{x1} \\ u_y &= u_{y1} - x\frac{\partial u_{x1}}{\partial y} + z\frac{\partial u_{z1}}{\partial y} \\ u_z &= u_{z1} \end{aligned} \tag{1}$$

where u_x, u_y and u_z are the displacement components of a point belonging to the beam domain along x, y and z, respectively; u_{x1}, u_{y1} and u_{z1} are the displacements of the beam axis; $-\partial u_{x1}/\partial y$ and $\partial u_{z1}/\partial y$ are the rotations of the cross-section about the z- (i.e. ϕ_z) and x-axis (i.e. ϕ_x). According to EBBM, the deformed cross-section remains plane and orthogonal to the beam axis because cross-sectional shear deformation phenomena are neglected. Shear stresses play a significant role in several problems (e.g., short beams and composite structures), and their neglect can lead to incorrect results. One may want to generalize Eq. (1) and overcome the EBBM assumption of the orthogonality of the cross-section. The improved displacement field results in the Timoshenko Beam Model (TBM),

$$\begin{aligned} u_x &= u_{x1} \\ u_y &= u_{y1} + x\phi_z - z\phi_x \\ u_z &= u_{z1} \end{aligned} \tag{2}$$

TBM constitutes an improvement over EBBM, because the cross-section does not necessarily remain perpendicular to the beam axis after deformation, and two degrees of freedom (i.e. the unknown rotations, ϕ_z and ϕ_x) are added to the original displacement field.

In this paper, particular attention is given to a 6-DOF model, i.e. a TBM model including torsion. The resulting kinematic foresees first-order shear effects and twisting.

$$\begin{aligned} u_x &= u_{x1} + z\phi_y \\ u_y &= u_{y1} + x\phi_z - z\phi_x \\ u_z &= u_{z1} - x\phi_y \end{aligned} \tag{3}$$

where ϕ_y represents the rigid rotation of the beam cross-section about the y-axis.

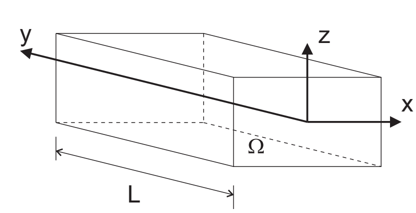

Fig. 1. Coordinate frame of the reference beam.

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