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A modified nonlinear fatigue damage accumulation model under multiaxial variable amplitude loading

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ABSTRACT

The purpose of this work is to develop a numerical tool, based on the finite element method, in order to determine the cumulative fatigue damage evolution and the remaining lifetime for mechanic parts under cyclic loadings. A modified nonlinear cumulative fatigue damage model, extended to multiaxial loading, taking into account the effects of the amplitude and sequence of variable amplitude loading is proposed. Various load histories were applied to a carbon steel type SM 45 C specimen. In order to highlight the ability of the new proposed model, the results were compared to those obtained according to the Palmgren–Miner's rule. The proposed model can be a reliable and useful tool for the lifetime prediction in the field of design and maintenance.

1. Introduction

Damage in mechanical structures is mainly the process of the initiation and growth of micro-cracks and cavities. A reliable lifetime prediction is particularly important in the design, safety assessments, and of engineering materials and structures. Manufacturing companies always search the means to find the defects of their machines and to determine the maintenance date in order to replace the damaged parts before their failure.

Indeed, it is important to formulate a method to evaluate the fatigue damage accumulation in order to predict the fatigue lifetime. Various theories and models have been proposed to predict damage accumulation in materials. Fatigue damage cumulative theories can be classified into two categories: linear [1–5] and nonlinear [6–16] damage models. An overview of some predictive models was given by Fatemi and Yang [17] and Schijve [18], with interesting discussions on influencing factors. Some models are based on energy equivalence [19–22]. Continuum Damage Mechanics (CDM) has provided an efficient nonlinear cumulative damage rule proposed by Rabotnov [8] and later developed by Chaboche [9]. The fatigue damage model of Lemaitre and Chaboche [12] is based on the macroscopic damage phenomena and applicable to many types of loading. Vaz et al. [23] present a numerical discussion of the coupled effects between ductile damage and temperature evolution. The effects of the average stress, the stress amplitude, and the nonlinear accumulation of the damage are also included in the (CDM) model. An elastoplastic constitutive model considering the damage, isotropic and kinematical hardening was presented by Kunc et al. [24]. A new approach suggested by Mesmacque et al. [13] called "Damaged Stress Model" does not require too many properties of material and it takes into account the history of loading.

Typical *S*–*N* curve, presented in Fig. 1, can be divided into three domains: Low Cycle Fatigue (LCF, $N = 10^4$–10^5 cycles), the High Cycle Fatigue (HCF, $10^5 \prec N \prec 10^7$ cycles) and the Very High Cycle Fatigue (VHCF, $N \succ 10^7$ cycles). In this work, the bound between High Cycle Fatigue (HCF) and Very High Cycle Fatigue (VHCF) is defined by the knee point on the *S*–*N* curve. The HCF region ranges from the number of cycle limit N_l to the upper number of cycle N_k corresponding to the knee point. The value of both N_l and N_k depends on the material (alloy), on its strength and also on the kind of loading. The *S*–*N* curves of steel are said to be asymptotic after the knee point N_k.

The purpose of this work is to develop a numerical tool, based on the new non-linear fatigue damage accumulation, in order to determine the damage evolution and the remaining lifetime for mechanic parts under multiaxial variable amplitude loading. In the present study, the loading is divided into two parts, i.e., elastic and cyclic. To derive the equivalent stress from the stress components, we use fatigue criteria.

In this paper a new proposed fatigue damage model is proposed in order to solve the problem of damage accumulation effects under fatigue loadings with several blocks. The new proposed fatigue

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Nomenclature

D_i	Damage at the ith load level
$\overrightarrow{F}_i^{(f)}$, F_i^a	Alternating vertical force and its amplitude, respectively
f_{-1}, t_{-1}	Fatigue limits in fully reversed bending and reversed torsion, respectively
HCF	High cycle fatigue
H-L	High-to-Low block-loading sequence
i	Load level
L	Distance between the direction of vertical force $\overrightarrow{F}_i^{(f)}$ and the left corner of bottom side
LCF	Low cycle fatigue
L-H	Low-to-High block-loading sequence
$\overrightarrow{M}_i^{(f)}$, M_i^a	Alternating bending moment and its amplitude, respectively
N_i	Number of cycles to failure (fatigue life) for the load level i
N_{i+1}^{RES}	Remaining number of cycles to failure at the $(i+1)$th load level
N_k	Knee point on the SN curve
n_i	Number of uniform cycles for the load level i
P_{max}^i	Maximum value the hydrostatic stress
S–N	Stress versus number of cycles
r_i	Cycle ratio corresponding to the ith load level
t	Time
U_X	Displacement along the x-axis
X, Y, Z	Axes
Σ_i^{eq}	Damage stress amplitude at the ith load level
$\overline{\Sigma}_{i+1}^{eq}$	Damage stress amplitude at the $(i+1)$th load level
α, β	Material parameters in Crosland criterion
κ, η and γ	Material parameters defining from the S–N curve
ν	Poison's ratio
ξ_a^i	Square root of the amplitude of the second invariant of the stress deviator tensor
$\sigma_{kl}^i(t)$	Stress components at the time t
$\sigma_{kl}^{i\,a}$, $\sigma_{kl}^{i\,m}$	Stress amplitudes and mean stresses, respectively
σ_{eq}^i, $\sigma_{eq}^{i\,max}$	Equivalent stress and maximum equivalent stress at each load level i, respectively
σ_i^m	Uniaxial tensile stress
σ_e, σ_u	Yield strength and ultimate stress, respectively
τ_u	Ultimate stress in torsion test
ω	Pulsation.

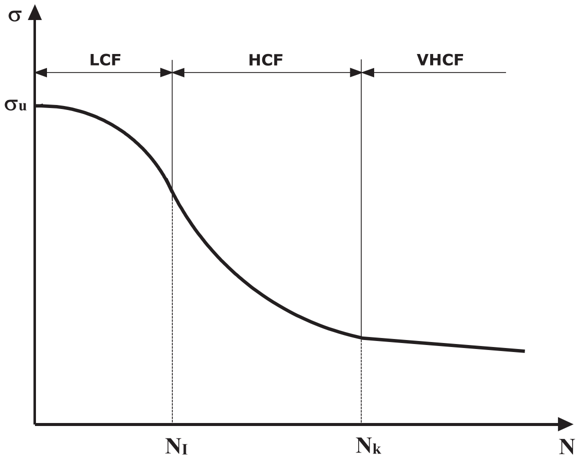

Fig. 1. Scheme of the S–N curve with the three regions: LCF, HCF and VHCF.

damage model is based on the previous work [13]. This new approach belongs to the category of the nonlinear damage cumulative models. The block loading data was transformed into an equivalent constant amplitude data, using the concept of maximum equivalent stress. An effort was made to get better life prediction via a an improved non-linear fatigue damage accumulation model. The application of the proposed model requires a very limited number of material properties obtained from monotonic and fatigue experiments. The present research paper is mainly devoted to investigate numerically how both parts (static and cyclic) of loadings and load sequences influence the cumulative damage and lifetime prediction. An example is given to demonstrate the reliability of the method. The main aim of this work is to provide a reliable and useful tool for the lifetime prediction in the field of design and maintenance.

2. Formulation of the proposed fatigue damage model

In order to determine the cumulative damage, it is necessary to replace the real sequence of cycles by an irregular loading with an assumed sequence of groups of uniform cycles. Each group,

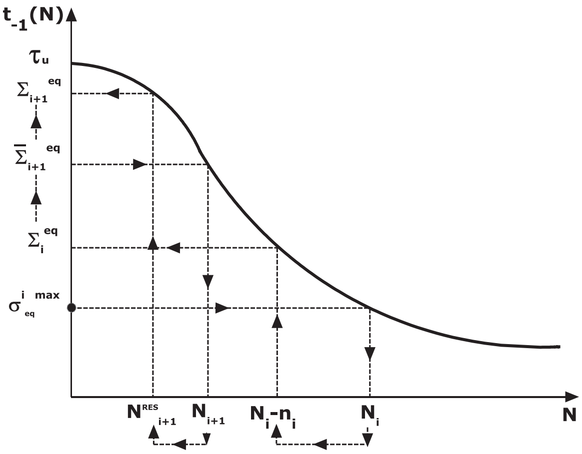

Fig. 2. Schematic illustration of the calculation of the cumulative damage and definition of the main parameters of the proposed model: $\sigma_{eq}^{i\,max}$, N_i, (N_i-n_i), $\overline{\Sigma}_{i+1}^{eq}$, N_{i+1}, Σ_i^{eq}, N_{i+1}^{RES} and Σ_{i+1}^{eq}.

comprising n_i number of uniform cycles, represents a corresponding load level i. At each load level i, a part is subjected to stress components $\sigma_{kl}^i(t)$ at the time t for n_i, while the theoretical life expectancy is N_i number of cycles. Each group of constant-amplitude loading is characterized by stress amplitudes $\sigma_{kl}^{i\,a}$ and mean stresses$\sigma_{kl}^{i\,m}$. At each point of a body subjected to any level of a sinusoidal synchronous loading, the stress components $\sigma_{kl}^i(t)$ at the time t can be expressed as follows:

$$\sigma_{kl}^i(t) = \sigma_{kl}^{i\,m} + \sigma_{kl}^{i\,a}\ \sin\ \omega t \tag{1}$$

where ω is the pulsation.

In this work, the analysis deals with replacement of the stress components $\sigma_{kl}^i(t)$ by one-dimensional equivalent stress σ_{eq}^i. In this work, the Crossland criterion has been chosen.

2.1. Determination of the equivalent loading σ_{eq}^i

Using the Crossland criterion [25] the multiaxial loading can be converted into an equivalent stress, which will be used to compute